QUIZ-1st 日河工 2B

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1) $y' = e^{-x}$

(1) It is an <u>*ordinary*</u> or <u>*partial*</u> differential equation of <u>*first*</u> or <u>*second*</u> order.
(pick up your answers, 5 scores for each)
Ans: It is an *ordinary* differential equation of *first* order. (see Textbook P.1~P.2)

(2) Try to get its general solution by direct integration (10 scores)

Ans: $\frac{dy}{dx} = e^{-x}$, $\int dy = \int e^{-x} dx$, ➔ $y = -e^{-x} + C$

(3) Verify your answer by substitution (10 scores)

Ans: $y' = \frac{dy}{dx} = \frac{d}{dx}(-e^{-x} + C) = e^{-x}$

(4) Try to get the particular solution corresponding to the initial solution y(0)=2
(10 scores)

Ans: $y(0) = -e^{-0} + C = -1 + C = 2$ ➔ $C = 3$

(5) Plot the graph of the particular solution, namely an *integral curve* of the equation
(20 scores) (hints: x=0, y=?; $x \to \infty$, $y \to ?$)

Ans: $y = -e^{-x} + 3$, $x = 0, y = 2$, $x \to \infty, y \to 3$,

$y = 0, -e^{-x} + 3 = 0$ ➔ $x = -\ln(3) \cong -1.1$

2) Verify the given function is a solution of the differential equation
(10 scores for each)

(1) $2yy' = 1$; $\varphi(x) = \sqrt{x-1}$ for $x > 1$

$$\varphi'(x) = \frac{1}{2}\frac{1}{\sqrt{x-1}}, \quad 2\varphi\varphi' = 2 \cdot \sqrt{x-1} \cdot \frac{1}{2}\frac{1}{\sqrt{x-1}} = 1$$

(2) $y' = -\frac{2y + e^x}{2x}$ for $x > 0$; $\varphi(x) = \frac{C - e^x}{2x}$

$$\varphi'(x) = -\frac{C - e^x}{2x^2} + \frac{-e^x}{2x} \;\rightarrow 2x\varphi' = -\frac{C - e^x}{x} - e^x = -2\varphi - e^x$$

(3) $y' + y = 2$; $\varphi(x) = 2 + ke^{-x}$

$$\varphi'(x) = -ke^{-x} \;\rightarrow\; \varphi'(x) + \varphi(x) = -ke^{-x} + 2 + ke^{-x} = 2$$

(4) $y' = \frac{y}{x} + 1$; $\varphi(x) = x\ln(x) + Cx$ for all $x > 0$

$$\varphi'(x) = \ln(x) + 1 + C, \rightarrow\; \varphi'(x) = \frac{x\ln(x) + Cx}{x} + 1 = \frac{\varphi(x)}{x} + 1$$

3) Verify by implicit differentiation that the given equation implicitly defines a solution of the differential equation (10 scores)

$$y^2 + xy - 2x^2 - 3x - 2y = C; \;\; y - 4x - 3 + (x + 2y - 2)y' = 0$$

$$d(y^2 + xy - 2x^2 - 3x - 2y)/dx = d(C)/dx$$

$$==> 2yy' + y + xy' - 4x - 3 - 2y' = 0$$

$$==> y - 4x - 3 + xy' + 2yy' - 2y' = 0$$

$$==> y - 4x - 3 + (x + 2y - 2)y' = 0$$