

QUIZ-2nd

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- 1) Verify the given function is a solution of the differential equation
(10 scores)

$$y' = \frac{y}{x} + 1; \quad \varphi(x) = x \ln(x) + Cx \quad \text{for all } x > 0$$

- 2) Verify by implicit differentiation that the given equation implicitly defines a solution of the differential equation (10 scores)

$$y^2 + xy - 2x^2 - 3x - 2y = C; \quad y - 4x - 3 + (x + 2y - 2)y' = 0$$

- 3) Solve the differential equation $3y' = 4x/y^2$ (15 scores)

- 4) Solve the differential equation $y' + y = \sin(x)$ (20 scores)

- 5) Solve the differential equation $\frac{2xy}{y-1} - y' = 0$ (15 scores)

- 6) For the differential equation $1 + (3x - e^{-2y})y' = 0$, (a) show that it is not exact, (b) find an integrating factor, (c) find the general solution (20 scores)

- 7) For a first-order differential equation, $y' = \frac{y}{x+y}$, check if it is homogeneous,

then find its general solution. (20 scores)

- 8) Consider the Riccati equation, $y' = \frac{1}{x}y^2 + \frac{1}{x}y - \frac{2}{x}$, with a solution $S(x) = 1$.

Try to get its general solution. *Hint*: Define a new variable z by setting

$$y = S(x) + \frac{1}{z} \quad (20 \text{ scores})$$