海洋大學河海工程學系九十四學年度 第一學期 ■期中 □小考 考試命題紙

考　試　科　目	開課系級	考試日期	印製份數	答案紙	命題教師	備　　　註
工程數學一	二 AB	1 月 19 日		■ 需 □不需	陳桂鴻 呂學育	期末考

學生可帶 □書本 ■計算機 □其他＿＿＿＿＿＿ ■皆不可 共 2 頁, 第 1 頁

1. $A=\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$.

(1) Find all eigenvalues and corresponding eigenvector. 3%

(2) Find generalized eigenvector and obtain the transition matrix P of A. 5%

(3) Find P^{-1}. 2%

(4) Find the Jordan canonical form of A by using the similar transform ($P^{-1}AP$). 5%

2. $A=\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

(1) Find eigenvectors and write the transition matrix P of A. 3%

(2) Find P^{-1} by using the orthogonal matrix property. 3%

(3) Find the diagonal form of A by using the similar transform ($P^{-1}AP$). 3%

(4) If $f(x)=x^{100}$, find the matrix $f(A)$ by using (a) the method of similar transform (matrix function), 9%
(b) Cayley-Hamilton theory. 9%

(5) Find A^{-1} by using (a) adjoint method, 2% (b) Cayley-Hamilton theory. 6%

3. For the given linear system

$$-x_1+3x_2 \quad = 0$$
$$x_1-2x_2+x_3=1$$
$$x_2+2x_3=0$$

we can rewrite it as a matrix-vector equation $AX=B$

with the matrix $A=\begin{pmatrix} -1 & 3 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, the vector $X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

(1) write out the vector B (1%)

(2) calculate det A (1%)

(3) is the matrix A nonsingular ? (1%)

(4) write out the adjoint of the matrix A (2%)

(5) find the inverse of the matrix A (2%)

(6) solve the system to give the vector X (2%)

(7) for the matrix A, what is the maximum number of independent column vectors? (1%)

(8) what is the rank of the matrix A ? (2%)

4. For a given matrix $A = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$

(1) find the eigenvalues (hint: with 1 as an eigenvalue of multiplicity 2) (2%)

(2) compute A^m; $m = 10$ by using Cayley-Hamilton theorem (9%)

(3) find a set of three mutually orthogonal eigenvectors (9%)

(4) use these vectors obtained in (3) to construct an orthogonal matrix that diagonalizes the matrix A (3%)

(5) compute A^m; $m = 10$ by diagonalizing the matrix A (5%)

5. For a given conic section of the form $2xy = 1$

(1) write the equation as the matrix product $X^T AX = 1$, with $X = \begin{pmatrix} x \\ y \end{pmatrix}$ (2%)

(2) eliminate the xy-term by means of an orthogonal matrix and diagonalization (8%)