

考試科目	開課系級	考試日期	印製份數	答案紙	命題教師	備註
工程數學一	二 A, B	10 月 24 日	111	<input checked="" type="checkbox"/> 需 <input type="checkbox"/> 不需	陳桂鴻 呂學育	第一次大考

1. A Bernoulli equation of  $xy' + y = \frac{1}{y}$ ,

(a) Linear or nonlinear (2%), why? (2%)

(b) Exact (Yes or No) (2%), why? (2%)

(c) Solve by Separable variable method. (6%)

(d) Solve by Linear O.D.E method (Convert the O.D.E to a linear equation by using the change of variable method). (7%)

(e) Solve by the Exact O.D.E method. (9%)

2. The Ricatti equation  $y' = y^2 - \frac{1}{x}y - \frac{4}{x^2}$  by using the solution  $y_2 = y_1 + \frac{1}{z}$  with  $y_1 = 2/x$ , we

obtain  $y_2 = \frac{2}{x} + \frac{1}{-\frac{1}{4}x + Cx^{-3}}$ ,  $C \in \mathbb{R}$ . By setting  $C = 0$ , we have  $y_2 = -\frac{2}{x}$ , solve  $y_3 = -\frac{2}{x} + \frac{1}{z}$ , please

find  $y_3$ . (10%)

3. Solve the singular solution and general solution of the Clairauts equation  $y = x \frac{dy}{dx} + f\left(\frac{dy}{dx}\right)$ , where

$f\left(\frac{dy}{dx}\right) = -e^{2y'}$ . (10%)

4. (a) is the differential equation  $(1-x)y' - 4x \sin(y) = \cos(x)$  linear or nonlinear in  $y$ ? (3%)

(b) is the differential equation  $(y^2 - 1)dx = xdy$  linear or nonlinear in  $x$ ? (3%)

(c) solve the separable differential equation  $\frac{dy}{dx} + \frac{y}{x} = 0$ ,  $y(1) = 1$  (5%)

(d) What is the slope of the tangent line to the graph of the solution  $y' = 6\sqrt{y} + 5x^3$  that through  $(-1, 1)$ ? (4%)

(e) Match the given differential equations with one or more of the solutions

(a)  $y = 0$ , (b)  $y = 2$ , (c)  $y = 2 + 2x^2$ , (d)  $y = 2x^2$ , (e)  $y = -2x^2$  (5%)

$x \frac{dy}{dx} = 2y$ ;  $\frac{dy}{dx} = 2y - 4$

5. Solve the initial-value problem  $\frac{dy}{dx} = (3x - y)^2 + 6x - 2y$ ,  $y(0) = -3$  (15%)

6.  $(2y \sin x - 3)dx - \cos x dy = 0$

(a) Is it exact? (2%) Why? (3%)

(b) Solve it by using an integrating factor. (10%)