

考試科目	開課系級	考試日期	印製份數	答案紙	命題教師	備註
工程數學(一)	二 A,B	11月18日	111	<input checked="" type="checkbox"/> 需 <input type="checkbox"/> 不需	陳桂鴻 呂學育	第二次期中考 共1頁

- (a) Determine the complementary solution, the particular solution and general solution of a nonhomogeneous linear DE $y^{(4)} + y''' = 1 - x^2 e^{-x}$. (10%)

(b) Find the general solution of $x^4 y'' + x^3 y' + 4x^2 y = 1$, given that $y_1(x) = x^2$ is a solution of the associated homogeneous equation. (5%)
- Cauchy-Euler equation $x^2 y'' + 10xy' + 8y = x^2$. Find the complementary solution by using

(a) Change of variable method ($t = \ln x$). (10%)

(b) Let $y = x^m$ and looks for m . (5%)

(c) If we has found one solution, $y_1(x) = x^{-1}$, find y_2 by using reduction order method. (10%)
- Solve the singular solution and general solution of the Clairauts equation $y = x \frac{dy}{dx} + f\left(\frac{dy}{dx}\right)$, where

$$f\left(\frac{dy}{dx}\right) = -e^{2y}. \quad (10\%)$$
- Consider (a), (b), (c), (d), determine whether the given set of function is linearly dependent or linearly independent on the interval $(0, \infty)$.

(a) $f_1(x) = \cos(2x)$, $f_2(x) = \sin(2x)$ (3%)

(b) $f_1(x) = e^{2x}$, $f_2(x) = e^{-2x}$ (3%)

(c) $f_1(x) = x^2$, $f_2(x) = x + 1$ (3%)

(d) $f_1(x) = 1 + x$, $f_2(x) = -x$, $f_3(x) = -x^2$ (3%)

(e) is the differential equation $(y^2 - 1)dx = xdy$ linear or nonlinear in x ? why? (3%)
- Given second-order differential equations on the interval $(-\infty, 0)$

(1) $x^2 y'' - 2xy' + 2y = 5x^3 \cos 3x$

(a) solve the associated homogeneous equation (y_c) (4%)

(b) find the particular solution of the nonhomogeneous equation (y_p) (6%)

(2) $x^2 y'' - 3xy' + 4y = 0$, $y(-1) = 2$, $y'(-1) = 4$. Find the solution of the differential equation (8%).
- Solve the given initial-value problem

$$\frac{d^2 x}{dt^2} + \beta \frac{dx}{dt} + 4x = \cos(2t); \quad x(0) = 0, x'(0) = 0$$

(a) if $\beta = 0$ find the solution of the differential equation (5%)

(b) if $\beta = 5$ find the solution of the differential equation (7%)

(c) if $t \rightarrow \infty$ compare your solutions a) and b) ($x \rightarrow \infty$?,...) (5%)