海洋大學河海工程學系九十四學年度 第一學期 ■期中 □小考 考試命題紙

考 試 科 目	開課系級	考試日期	印製份數	答案紙	命題教師	備 註
工程數學一	二 A, B	10 月 24 日	111	■需 □不需	陳桂鴻 呂學育	第一次大考

1. A Bernoulli equation of $xy' + y = \frac{1}{y}$,

(a) Linear or nonlinear (2%), why? (2%)

(b) Exact (Yes or No) (2%), why? (2%)

(c) Solve by Separable variable method. (6%)

(d) Solve by Linear O.D.E method (Convert the O.D.E to a linear equation by using the change of variable method). (7%)

(e) Solve by the Exact O.D.E method. (9%)

Ans: (a) nonlinear, 有因變數的二次項.

(b) nonexact, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$.

(c) $\frac{dy}{dx} = (\frac{1-y^2}{y})\frac{1}{x}$, $\int \frac{y}{1-y^2}dy = \int \frac{1}{x}dx$, $\frac{-1}{2}\ln|1-y^2| = \ln|x| + c$, $(1-y^2)^{-\frac{1}{2}} = x + c_1$.

(d) $yy' + \frac{1}{x}y^2 = \frac{1}{x}$, **Let** $u = y^2$, $u' = 2yy'$, $\therefore \frac{1}{2}u' + \frac{1}{x}u = \frac{1}{x}$, $-\frac{1}{2}\ln|1-u| = \ln|x| + \ln|c|$,

$(1-y^2)^{-\frac{1}{2}} = cx$.

(e) $(y^2-1)dx + xydy = 0$, $\frac{\partial \mu M}{\partial y} = \frac{\partial \mu N}{\partial x}$, $(y^2-1)\frac{\partial \mu}{\partial y} + 2\mu y = xy\frac{\partial \mu}{\partial x} + \mu y$; **Let** $\mu = \mu(x)$, $\therefore \mu = x$

$\frac{\partial \phi}{\partial x} = \mu M$, $\therefore \phi = \frac{1}{2}x^2y^2 - \frac{1}{2}x^2 + h(y)$; $\frac{\partial \phi}{\partial y} = \mu N$, $x^2y + h'(y) = x^2y$, $\therefore h(y) = c$

$\therefore \frac{1}{2}x^2y^2 - \frac{1}{2}x^2 + c = 0$.

2.The Ricatti equation $y' = y^2 - \frac{1}{x}y - \frac{4}{x^2}$ **by using the solution** $y_2 = y_1 + \frac{1}{z}$ **with** $y_1 = 2/x$, **we obtain** $y_2 = \frac{2}{x} + \frac{1}{-\frac{1}{4}x + Cx^{-3}}$, $C \in \Re$. **By setting** $C = 0$, **we have** $y_2 = -\frac{2}{x}$, **solve** $y_3 = -\frac{2}{x} + \frac{1}{z}$, **please find** y_3. **(10%)**

Ans: $y_3 = -\frac{2}{x} + \frac{1}{z}$, $y_3{}' = \frac{2}{x^2} - \frac{1}{z^2}z'$, $\frac{2}{x^2} - \frac{1}{z^2}z' = (\frac{-2}{x} + \frac{1}{z})^2 - \frac{1}{x}(\frac{-2}{x} + \frac{1}{z}) - \frac{4}{x^2}$, $\therefore z' - \frac{5}{x}z = -1$

$I = x^{-5}$, $x^{-5}z' - 5x^{-6}z = -x^{-5}$, $x^{-5}z = \frac{1}{4}x^{-4} + c$, $\therefore z = \frac{x}{4} + cx^5$, $y_3 = \frac{-2}{x} + \frac{1}{\frac{x}{4} + cx^5}$.

3. Solve the singular solution and general solution of the Clairauts equation $y = x\frac{dy}{dx} + f(\frac{dy}{dx})$**, where** $f(\frac{dy}{dx}) = -e^{2y'}$**. (10%)**

Ans: $y = xy' - e^{2y'}$**, Let** $p = y'$**,** $\therefore y = xp - e^{2p}$**,** $y' = p + xp' - 2p'e^{2p}$**,** $\therefore p'(x - 2e^{2p}) = 0$

general solution: $\begin{cases} p' = 0 \\ y = xp - e^{2p} \end{cases}$**,** $p = c$**,** $\therefore y = cx - e^{2c}$

singular solution: $\begin{cases} x - 2e^{2p} = 0 \\ y = xp - e^{2p} \end{cases}$**,** $p = \frac{1}{2}\ln\left|\frac{x}{2}\right|$**,** $\therefore y = \frac{1}{2}x\ln\left|\frac{x}{2}\right| - \frac{x}{2}$**.**

4. (a) is the differential equation $(1-x)y' - 4x\sin(y) = \cos(x)$ **linear or nonlinear in** y**? (3%)**

Ans: nonlinear.

(b) is the differential equation $(y^2 - 1)dx = xdy$ **linear or nonlinear in** x**? (3%)**

Ans: linear.

(c) solve the separable differential equation $\frac{dy}{dx} + \frac{y}{x} = 0, \ y(1) = 1$ **(5%)**

Ans: $\frac{dy}{y} = -\frac{dx}{x}$

$\ln|y| = \ln|x^{-1}| + \ln|c| \rightarrow y = cx^{-1}$

$y(1) = 1 \rightarrow y = x^{-1}$.

(d) What is the slope of the tangent line to the graph of the solution $y' = 6\sqrt{y} + 5x^3$ **that through (-1, 1) ? (4%)**

Ans: The slope of the tangent line ： $y' = 6\sqrt{1} + 5(-1)^3 = 1$**.**

(e) Match the given differential equations with one or more of the solutions

(a) $y = 0$**, (b)** $y = 2$**, (c)** $y = 2 + 2x^2$**, (d)** $y = 2x^2$**, (e)** $y = -2x^2$ **(5%)**

$x\frac{dy}{dx} = 2y$ ； $\frac{dy}{dx} = 2y - 4$

Ans: $x\frac{dy}{dx} = 2y \Rightarrow (a),(d),(e)$ **,** $\frac{dy}{dx} = 2y - 4 \Rightarrow (b),(c)$.

5. Solve the initial-value problem $\frac{dy}{dx} = (3x - y)^2 + 6x - 2y, \ y(0) = -3$ **(15%)**

Ans: Let $u=3x-y \quad \rightarrow \quad \frac{dy}{dx}=3-\frac{du}{dx}$

$$\frac{du}{dx}=-u^2-2u+3=-(u+3)(u-1)$$

$$\frac{du}{-(u+3)(u-1)}=dx$$

$$\frac{1}{4}\int(\frac{1}{u+3}-\frac{1}{u-1})du=\int dx$$

$$\ln\left|(\frac{u+3}{u-1})^{\frac{1}{4}}\right|=x+c$$

$$e^{4x+c}=\frac{u+3}{u-1} \quad \rightarrow \quad e^{4x+c}=\frac{3x-y+3}{3x-y-1}$$

$$y(0)=-3 \quad \rightarrow \quad c=\ln|3|.$$

The solution of the initial-value problem is $3e^{4x}=\frac{3x-y+3}{3x-y-1}$.

6. $(2y\sin x-3)dx-\cos xdy=0$

(a) Is it exact? (2%) Why? (3%)

Ans: It is not exact.

$$\frac{\partial M}{\partial y}=2\sin x \neq \frac{\partial N}{\partial x}=\sin x \quad .$$

(b) Solve it by using an integrating factor. (10%)

Ans: Let $f(x)=\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=-\frac{\sin x}{\cos x}$

$$\rightarrow \quad \mu(x)=e^{\int f(x)dx}=\cos x$$

$$(2y\sin x\cos x-3\cos x)dx-\cos^2 xdy=0$$

$$\int(2y\sin x\cos x-3\cos x)dx=-\frac{1}{2}y\cos 2x-3\sin x+f(y)$$

$$\int -\cos^2 xdy=-\frac{1}{2}y\cos 2x-\frac{y}{2}+g(x)$$

$$\rightarrow \quad \frac{1}{2}y\cos 2x+3\sin x+\frac{y}{2}=c.$$