Find a second-order differential equation having the function as general solution

a) $c_1 e^{-2x} + c_2 e^{3x}$

➔ the roots of the *characteristic equation* are $\lambda_1 = -2, \quad \lambda_2 = 3$

➔ the *characteristic equation* is $(\lambda+2)(\lambda-3)=0, \quad \lambda^2 - \lambda - 6 = 0$

➔the differential equation is $y'' - y' - 6y = 0$

b) $c_1 e^{-3x}\cos(2x) + c_2 e^{-3x}\sin(2x)$

➔ $c_1 e^{-3x}\cos(2x) + c_2 e^{-3x}\sin(2x) = e^{-3x}\left[c_1\cos(2x) + c_2\sin(2x)\right]$

➔ the roots of the *characteristic equation* are $\lambda_1 = -3+2i, \quad \lambda_2 = -3-2i$

➔ the *characteristic equation* is $(\lambda+3-2i)(\lambda+3+2i)=0, \quad \lambda^2 + 6\lambda + 13 = 0$

➔the differential equation is $y'' + 6y' + 13y = 0$

c) $c_1 e^{-4x} + c_2 x e^{-4x}$

➔ the roots of the *characteristic equation* are $\lambda_1 = -4, \quad \lambda_2 = -4$

➔ the *characteristic equation* is $(\lambda+4)(\lambda+4)=0, \quad \lambda^2 + 8\lambda + 16 = 0$

➔the differential equation is $y'' + 8y' + 16y = 0$