HOMEWORK #11 (Chapter 8 Exercises--- Matrices)

**For yourself**

1) Verify that the given matrix satisfies its own characteristic equation

$$A = \begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix} \quad \text{(page 403, Problem 1)}$$

2) Use the method of section to compute $A^m$. Use this result to compute the indicated power of the matrix $A$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}; \quad m = 10 \quad \text{(page 403, Problem 7)}$$

3) Show that the given matrix has an eigenvalue $\lambda_1$ of multiplicity two. As a consequence, the equations $\lambda^m = c_0 + c_1\lambda_1$ does not yield enough independent equations to form a system for determining the coefficients $c_0$, $c_1$. Use the derivative (with respect to $\lambda$) of the equation evaluated at $\lambda_1$ as the extra needed equation to form a system. Compute $A^m$ and use this result to compute the indicated power of the matrix $A$

$$A = \begin{pmatrix} 7 & 3 \\ -3 & 1 \end{pmatrix}; \quad m = 6 \quad \text{(page 403, Problem 11)}$$

4) Proceed as in Example 3 to construct an orthogonal matrix from the eigenvectors of the given symmetric matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \text{(page 410, Problem 15)}$$

5) Use Theorem 8.29 to find values of $a$ and $b$ so that the given matrix is orthogonal

$$A = \begin{pmatrix} 3/5 & a \\ 4/5 & b \end{pmatrix} \quad \text{(page 410, Problem 19)}$$

6) (page 410, Problem 21)

a) Verify that

$$K_1 = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, \text{ and } K_3 = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$$

are eigenvectors for the symmetric matrix

$$A = \begin{pmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{pmatrix}$$

corresponding to the eigenvalues $\lambda_1 = 9, \lambda_2 = \lambda_3 = -9$ , respectively

b) Find a set of three mutually orthogonal eigenvectors for the matrix A in part a)