HOMEWORK #1s (Chapter 1 Review Exercises)

1) $y = c_1 e^x + c_2 x e^x$ (Problem 5.)

Compute y' and y'' and then combine these results as a linear second-order differential equation that is free of the symbols c_1 and c_2 and has the form $F(y, y', y'') = 0$. The symbols c_1 and c_2 represent constants.

ANS $y' = c_1 e^x + c_2 x e^x + c_2 e^x$; $y'' = c_1 e^x + c_2 x e^x + 2c_2 e^x$;

$y'' + y = 2(c_1 e^x + c_2 x e^x) + 2c_2 e^x = 2(c_1 e^x + c_2 x e^x + c_2 e^x) = 2y'$;

$y'' - 2y' + y = 0$

In Problems 2)~4), match each of the given differential equations with one or more of the solutions:
(a) $y = 0$ (b) $y = 2$ (c) $y = 2x$ (d) $y = 2x^2$

2) $xy' = 2y$ (Problem 7.)

ANS a,d

3) $y' = 2y - 4$ (Problem 9.)

ANS b

4) $y'' + 9y = 18$ (Problem 11.)

ANS b

5) What is the slope of the tangent line to the graph of the solution $y' = 6\sqrt{y} + 5x^3$ that through (-1, 4) ? (Problem 20.)

ANS The slope of the tangent line is $y'\big|_{(-1,4)} = 6\sqrt{4} + 5(-1)^3 = 7$

6) Verify that the indicated function is a particular solution of the given differential equation. Given an interval of definition ***I*** for the solution.

$x^2 y'' + xy' + y = 0;\ \ y = \sin(\ln x)$

ANS Differentiating $y = \sin(\ln x)$ we obtain $y' = \cos(\ln x)/x$ and

$y'' = -\left[\sin(\ln x) + \cos(\ln x)\right]/x^2$.

Then $x^2 y'' + xy' + y = x^2\left(-\dfrac{\sin(\ln x) + \cos(\ln x)}{x^2}\right) + x\dfrac{\cos(\ln x)}{x} + \sin(\ln x) = 0$

Then interval of definition is $x > 0$