HOMEWORK #2s (Chapter 2 Exercises---Separable Variables, Linear Equations)

1) Solve $y' = \frac{3x-y-9}{x+y+1}$ (O'Neil p.46, problem 17)

ANS $3(x-2)^2 - 2(x-2)(y+3) - (y+3)^2 = c$

2) Solve $y' = \frac{x-2y}{3x-6y+4}$ (O'Neil p.47, problem 24)

ANS $3(x-2y) - \ln(x-2y+4)^8 = x + c$

Consider 3)~4), find the general solution of the given differential equation. Give the largest interval over which the general solution is defined. Determine whether there are any transient terms (y_c, see page 51 of the textbook) in the general solution.

3) $x\frac{dy}{dx} - y = x^2\sin(x)$ (page 57, Problem 9.)

ANS For $y' - \frac{1}{x}y = x\sin x$ an integrating factor is $e^{-\int \frac{1}{x}dx} = \frac{1}{x}$ so that $\frac{d}{dx}[\frac{1}{x}y] = \sin x$ and $y = cx - x\cos x$ for $0 < x < \infty$

4) $ydx - 4(x+y^6)dy = 0$ (page 57, Problem 15.)

ANS For $\frac{dx}{dy} - \frac{4}{y}x = 4y^5$ an integrating factor is $e^{-\int \frac{4}{y}dy} = y^{-4}$ so that $\frac{d}{dy}[y^{-4}x] = 4y$ and $x = 2y^6 + cy^4$ for $0 < y < \infty$

5) Solve the given Bernoulli equation by using an appropriate substitution.

$t^2\frac{dy}{dt} + y^2 = ty$ (page 67, Problem 19.)

ANS From $y' - \frac{1}{t}y = -\frac{1}{t^2}y^2$ and $\omega = y^{-1}$ we obtain $\frac{d\omega}{dt} + \frac{1}{t}\omega = \frac{1}{t^2}$. An integrating factor is t so that $t\omega = \ln t + c$ or $y^{-1} = \frac{1}{t}\ln t + \frac{c}{t}$. Writing this in the form $\frac{t}{y} = \ln x + c$, we see that the solution can also be expressed in the form $e^{\frac{t}{y}} = c_1 x$.

6) Find a one-parameter family of solutions for the differential equation

$\frac{dy}{dx} = -\frac{4}{x^2} - \frac{1}{x}y + y^2$ where $y_1=2/x$ is a known solution of the equation

(page 68, Problem 33(b).)

ANS Identify $P(x) = \frac{-4}{x^2}, Q(x) = \frac{-1}{x}$, and $R(x) = 1$.Then $\frac{d\omega}{dx} + (-\frac{1}{x} + \frac{4}{x})\omega = -1$. An integrating factor is x^3 so that $x^3\omega = -\frac{1}{4}x^4 + c$ or $u = [-\frac{1}{4}x + cx^{-3}]^{-1}$. Thus, $y = \frac{2}{x} + u$.