HOMEWORK #3s (Chapter 2 Exercises---Exact Equations, Homogeneous Equations)

Consider 1) , determine whether the given differential equation is exact. If exact, solve it.

1) $(\tan x - \sin x \sin y)dx + \cos x \cos y dy = 0$ (page 63, Problem 17)

ANS Let $M = \tan x - \sin x \sin y$ and $N = \cos x \cos y$ so that $M_y = -\sin x \cos y = N_x$.

From $f_x = \tan x - \sin x \sin y$ we obtain $f = \ln|\sec x| + \cos x \sin y + h(y), h'(y) = 0,$ and

$h(y) = 0$. The solution is $\ln|\sec x| + \cos x \sin y = c$.

2) Solve the given initial-value problem

$(y^2 \cos x - 3x^2 y - 2x)dx + (2y \sin x - x^3 + \ln y)dy = 0 \quad y(0) = e$ (page 63, problem 25)

ANS Let $M = y^2 \cos x - 3x^2 y - 2x$ and $N = 2y \sin x - x^3 + \ln y$ so that

$M_y = 2y \cos x - 3x^2 = N_x$. From $f_x = y^2 \cos x - 3x^2 y - 2x$ we obtain

$f = y^2 \sin x - x^3 y - x^2 + h(y), h'(y) = \ln y,$ and $h(y) = y \ln y - y$. The general solution is $y^2 \sin x - x^3 y - x^2 + y \ln y - y = c$. If $y(0) = e$ then $c = 0$ and the solution of the initial-value problem is $y^2 \sin x - x^3 y - x^2 + y \ln y - y = 0$.

Consider 3)~4), solve the given differential equation by finding an appropriate integrating factor.

3) $(2y^2 + 3x)dx + 2xydy = 0$ (page 63, Problem 31)

ANS We note that $(M_y - N_x)/N = 1/x,$ so an integrating factor is $e^{\int dx/x} = x$. Let

$M = 2xy^2 + 3x^2$ and $N = 2x^2 y$ so that $M_y = 4xy = N_x$. From $f_x = 2xy^2 + 3x^2$ we

obtain $f = x^2 y^2 + x^3 + h(y), h'(y) = 0,$ and $h(y) = 0$. The solution of the differential equation is $x^2 y^2 + x^3 = c$.

4) $6xydx + (4y + 9x^2)dy = 0$ (page 63, Problem 33)

ANS We note that $(N_x - M_y)/M = 2/y,$ so an integrating factor is $e^{\int 2dy/y} = y^2$. Let

$M = 6xy^3$ and $N = 4y^3 + 9x^2 y^2$ so that $M_y = 18xy^2 = N_x$. From $f_x = 6xy^3$ we obtain

$f = 3x^2 y^3 + h(y), h'(y) = 4y^3,$ and $h(y) = y^4$. The solution of the differential equation is $3x^2 y^3 + y^4 = c$.

5) Solve the given homogeneous equation by using an appropriate substitution.

$-ydx + \left(x + \sqrt{xy}\right)dy = 0$ (page 67, Problem 9)

ANS Letting $y = ux$ we have

$$-uxdx + (x + \sqrt{u}x)(udx + xdu) = 0$$
$$(x + x\sqrt{u})du + u^{3/2}dx = 0$$
$$(u^{-3/2} + \frac{1}{u})du + \frac{dx}{x} = 0$$
$$-2u^{-1/2} + \ln|u| + \ln|x| = c$$
$$\ln|y/x| + \ln|x| = 2\sqrt{x/y} + c$$
$$y(\ln|y| - c)^2 = 4x$$

6) Solve the given initial-value problem.

$\left(x + ye^{y/x}\right)dx - xe^{y/x}dy = 0 \quad y(1) = 0$ (page 67, Problem 13)

ANS Letting $y = ux$ we have

$$(x + uxe^u)dx - xe^u(udx + xdu) = 0$$
$$dx - xe^u du = 0$$
$$\frac{dx}{x} - e^u du = 0$$
$$\ln|x| - e^u = c$$
$$\ln|x| - e^{y/x} = c$$

Using $y(1) = 0$ we find $c_1 = -1$. The solution of the initial-value problem is $\ln|x| = e^{y/x} - 1$.