HOMEWORK #4s (Chapter 3 Exercises--- Preliminary Theory, Reduction of Order, Homogeneous Linear Equations with Constant Coefficients)

Consider 1), 2), determine whether the given set of function is linearly dependent or linearly independent on the interval $(-\infty, \infty)$.

1) $f_1(x) = x,\ \ f_2(x) = x^2,\ \ f_3(x) = 4x - 3x^2$ (page 113, Problem 15)

ANS Since $(-4)x + (3)x^2 + (1)(4x - 3x^2) = 0$ the functions are linearly dependent.

2) $f_1(x) = 1 + x,\ \ f_2(x) = x,\ \ f_3(x) = x^2$ (page 113, Problem 21)

ANS The functions are linearly independent since $W(1+x, x, x^2) = \begin{vmatrix} 1+x & x & x^2 \\ 1 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2 \neq 0$.

3) Verify that the given function form a fundamental set of the differential equation on the indicated interval. Form the general solution. $x^2 y'' - 6xy' + 12y = 0;\ \ x^3, x^4, (0, \infty)$

(page113, problem 27)

ANS The functions satisfy the differential equation and are linearly independent since $W(x^3, x^4) = x^6 \neq 0$, for $0 < x < \infty$. The general solution is $y = c_1 x^3 + c_2 x^4$.

Consider 4)~ 5), the indicated function y_1 is a solution of the given equation. Use reduction of order to find a second solution y_2.

4) $9y'' - 12y' + 4y = 0;\ \ y_1 = e^{2x/3}$ (page 116, Problem 7)

ANS Define $y = u(x)e^{2x/3}$ so $y' = \frac{2}{3}e^{2x/3}u + e^{2x/3}u'$, $y'' = e^{2x/3}u'' + \frac{4}{3}e^{2x/3}u' + \frac{4}{9}e^{2x/3}u$ and

$9y'' - 12y' + 4y = 9e^{2x/3}u'' = 0$. Therefore $u'' = 0$ and $u = c_1 x + c_2$. Taking $c_1 = 1$ and

$c_2 = 0$ we see that a second solution is $y_2 = xe^{2x/3}$.

5) $x^2y'' - 7xy' + 16y = 0$; $y_1 = x^4$ (page 116, Problem 9)

ANS Identifying $P(x) = -7/x$ we have $y_2 = x^4 \int \frac{e^{-\int -(7/x)dx}}{x^8} dx = x^4 \int \frac{1}{x} dx = x^4 \ln|x|$. A second solution is $y_2 = x^4 \ln|x|$.

Consider 6)~ 7), find the general solution of the given second-order differential equation.

6) $y'' - y' - 6y = 0$

(page 122, Problem 3)

ANS From $m^2 - m - 6 = 0$ we obtain $m = 3$ and $m = -2$ so that $y = c_1e^{3x} + c_2e^{-2x}$.

7) $y'' + 8y' + 16y = 0$

(page 122, Problem 5)

ANS From $m^2 + 8m + 16 = 0$ we obtain $m = -4$ and $m = -4$ so that $y = c_1e^{-4x} + c_2xe^{-4x}$.