HOMEWORK #5s (Chapter 3 Exercises--- Homogeneous Linear Equations with Constant Coefficients, Undetermined Coefficients)

1) Find the general solution of the given second-order differential equation.

$3y'' + 2y' + y = 0$ (page 122, Problem 13)

ANS From $3m^2 + 2m + 1 = 0$ we obtain $m = -1/3 \pm \sqrt{2}\, i/3$ so that

$y = e^{-x/3}(c_1 \cos\sqrt{2}x/3 + c_2 \sin\sqrt{2}x/3)$.

2) Find the general solution of the given higher-order differential equation.

$y''' + 3y'' + 3y' + y = 0$ (page 122, Problem 21)

ANS From $m^3 + 3m^2 + 3m + 1 = 0$ we obtain $m = -1$, $m = -1$, and $m = -1$ so that

$y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}$.

3) Solve the given initial-value problem.

$y''' + 12y'' + 36y' = 0;\ y(0) = 0, y'(0) = 1, y''(0) = -7$

(page122 problem 35)

ANS From $m^3 + 12m^2 + 36m = 0$ we obtain $m = 0$, $m = -6$, and $m = -6$ so that

$y = c_1 + c_2 e^{-6x} + c_3 x e^{-6x}$. If $y(0) = 0$, $y'(0) = 1$, and $y''(0) = -7$ then $c_1 + c_2 = 0$,

$-6c_2 + c_3 = 1$, $36c_2 - 12c_3 = -7$, so $c_1 = 5/36$, $c_2 - 5/36$, $c_3 = 1/6$, and

$y = \frac{5}{36} - \frac{5}{36} e^{-6x} + \frac{1}{6} x e^{-6x}$.

4) Solve the given differential equation by undetermined coefficients.

$y'' + 2y' + y = \sin(x) + 3\cos(2x)$ (page 131, Problem 19)

ANS From $m^2 + 2m + 1 = 0$ we find $m_1 = m_2 = -1$. Then $y_c = c_1 e^{-x} + c_2 x e^{-x}$ and we

assume $y_p = A\cos x + B\sin x + C\cos 2x + D\sin 2x$. Substituting into the differential

equation we obtain $2B = 0$, $-2A = 1$, $-3C + 4D = 3$, and $-4C - 3D = 0$. Then $A = -\frac{1}{2}$,

$B = 0$, $C = -\frac{9}{25}$, $D = \frac{12}{25}$, $y_p = -\frac{1}{2}\cos x - \frac{9}{25}\cos 2x + \frac{12}{25}\sin 2x$, and

$y = c_1 e^{-x} + c_2 x e^{-x} - \frac{1}{2}\cos x - \frac{9}{25}\cos 2x + \frac{12}{25}\sin 2x$.

5) Solve the given initial-value problem

$$\frac{d^2x}{dt^2}+\omega^2 x = F_0 \sin(\omega t); \;\; x(0)=0, x'(0)=0 \quad \text{(page 131, Problem 33)}$$

ANS We have $x_c = c_1 \cos\omega t + c_2 \sin\omega t$ and we assume $x_p = At\cos\omega t + Bt\sin\omega t$.

Substituting into the differential equation we find $A = -F_0/2\omega$ and $B=0$. Thus $x = c_1\cos\omega t + c_2\sin\omega t - (F_0/2\omega)t\cos\omega t$. From the initial conditions we obtain $c_1 = 0$ and $c_2 = F_0/2\omega^2$, so $x = (F_0/2\omega^2)\sin\omega t - (F_0/2\omega)t\cos\omega t$.

6) Solve the given initial-value problem

$$y''' - 2y'' + y' = 2 - 24e^x + 40e^{5x}; \;\; y(0)=\frac{1}{2}, y'(0)=\frac{5}{2}, y''(0) = -\frac{9}{2}$$

(page 131, Problem 35)

ANS We have $y_c = c_1 + c_2 e^x + c_3 x e^x$ and we assume $y_p = Ax + Bx^2e^x + Ce^{5x}$. Substituting into the differential equation we find $A=2$, $B=-12$, and $C=\frac{1}{2}$. Thus $y = c_1 + c_2e^x + c_3xe^x + 2x - 12x^2e^x + \frac{1}{2}e^{5x}$. From the initial conditions we obtain $c_1 = 11$, $c_2 = -11$, and $c_3 = 9$, so $y = 11 - 11e^x + 9xe^x + 2x - 12x^2e^x + \frac{1}{2}e^{5x}$.