HOMEWORK #8 (Chapter 5 Exercises--- Solutions about Singular Points)

Due on Dec. 7

1) Determine the singular points of the given differential equation. Classify each singular point as regular or irregular.

$x^3(x^2-25)(x-2)^2 y'' + 3x(x-2)y' + 7(x+5)y = 0$ (page 254, Problem 9)

2) Put the given differential equation into the form (3), *on page 247 of the textbook*, for each regular singular point of the equation. Identify the functions $p(x)$ and $q(x)$.

$(x^2-1)y'' + 5(x+1)y' + (x^2-x)y = 0$ (page 254, Problem 11)

3) In this problem, $x = 0$ is a regular singular point of the given differential equation. Use the general form of the indicial equation in (14), *on page 251 of the textbook*, to find the indicial roots of the singularity. Without solving, discuss the number of series solutions you would expect to find using the method of Frobenius.

$x^2 y'' + (\frac{5}{3}x + x^2)y' - \frac{1}{3}y = 0$ (page 255, Problem 13)

4) In this problem, $x = 0$ is a regular singular point of the given differential equation. Show that the indicial roots of the singularity do not differ by an integer. Using the method of Frobenius to obtain two linearly independent series solutions about $x = 0$. Form the general solution on $(0, \infty)$. $9x^2 y'' + 9x^2 y' + 2y = 0$ (page 255, Problem 23)

In this **problem5), 6)**, $x = 0$ is a regular singular point of the given differential equation. Show that the indicial roots of the singularity differ by an integer. Using the method of Frobenius to obtain at least one series solutions about $x = 0$. Form the general solution on $(0, \infty)$.

5) $xy'' + 2y' - xy = 0$ (page 255, Problem 25)

6) $xy'' - xy' + y = 0$ (page 255, Problem 27)

7) In this problem, $x = 0$ is a regular singular point of the given differential equation. Show that the indicial roots of the singularity differ by an integer. Use the recurrence relation found by the method of Frobenius first with the largest root r_1. How many solutions did you find ? Next use the recurrence relation with the smaller root r_2. How many solutions did you find ? $xy'' + (x-6)y' - 3y = 0$ (page 255, Problem 31)