

HOMEWORK #9 (Chapter 5 Exercises--- Two Special Equations)

Due on Dec. 14

1) Find the general solution of the given differential equation on $(0, \infty)$

$$4x^2 y'' + 4xy' + (4x^2 - 25)y = 0 \quad (\text{page 264, Problem 3})$$

a) rewrite the given DE into the standard form of Bessel's equation

b) identify the value of ν

c) write out the Bessel function of the first kind of order ν and $-\nu$

d) are your J_ν and $J_{-\nu}$ linearly independent? why?

e) write out the general solution of the given differential equation on $(0, \infty)$

2) Find the general solution of the given differential equation on $(0, \infty)$

$$xy'' + y' + xy = 0 \quad (\text{page 264, Problem 5})$$

a) rewrite the given DE into the standard form of Bessel's equation

b) identify the value of ν

c) write out the Bessel function of the first kind of order ν and $-\nu$

d) are your J_ν and $J_{-\nu}$ linearly independent? why?

e) write out the Bessel function of the second kind of order ν

f) write out the general solution of the given differential equation on $(0, \infty)$

3) Find the general solution of the given differential equation on $(0, \infty)$

$$x^2 y'' + xy' + (9x^2 - 4)y = 0 \quad (\text{page 264, Problem 7})$$

a) write out the general solution of $x^2 y'' + xy' + (x^2 - 4)y = 0$

b) by referring to $x^2 y'' + xy' + (\lambda^2 x^2 - 4)y = 0$, identify the value of ν

c) write out the general solution of $x^2 y'' + xy' + (9x^2 - 4)y = 0$ on $(0, \infty)$

4) Legendre's equation and Legendre polynomials (page 265, Problem 35)

- a) write out the standard form of the Legendre's equation
- b) write out the general solution of the Legendre's equation
- c) explain what the Legendre polynomials is
- d) write out the Legendre polynomials $P_5(x)$, $P_6(x)$
- e) write the differential equation for which $P_5(x)$ is a particular solution
- f) write the differential equation for which $P_6(x)$ is a particular solution