Review Problems for Final Exam

Jan. 2006

1) For the given matrix $A = \begin{pmatrix} 2 & 4 & 7 \\ 6 & 0 & 3 \\ 1 & 5 & 3 \end{pmatrix}$

(1) what is the size of the matrix A ?

(2) is it a square matrix of order **3 ?**

(3) is it a symmetrix matrix ?

(4) write out all the main diagonal entries ?

(5) calculate the minor determinant M_{12}

(6) calculate the cofactor of a_{12}

(7) calculate det A

(8) write out the transpose of the matrix A

(9) write out the adjoint of the matrix A

(10) find the inverse of the matrix A

(11) is the matrix A nonsingular (or invertible)? why ?

(12) what is the maximum number of independent column vectors ?

(13) what is the maximum number of independent row vectors ?

(14) is the rank of the matrix A equal to 3 ? why ?

(15) does the matrix A satisfy Cayley-Halmilton theory ? why ?

2) For a given matrix $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

(1) find the eigenvalues and eigenvectors

(2) compute A^m; $m = 10$ by using Cayley-Halmilton theory

(3) compute A^m; $m = 10$ by diagonalizing the matrix A (page 426, Problem 37)

(4) you should also try $A = \begin{pmatrix} 7 & 3 \\ -3 & 1 \end{pmatrix}$; $A = \begin{pmatrix} -2 & 2 & -1 \\ 2 & 1 & -2 \\ -3 & -6 & 0 \end{pmatrix}$

3) For the given matrix $A = \begin{pmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ (example 1, page 406)

(1) is it a symmetric matrix with real entries ?

(2) find the eigenvalues, are all the eigenvalues real ?

(3) find the eigenvectors, are the eigenvectors corresponding to distinct eigenvalues orthogonal ?

(4) what is a orthogonal matrix ?

(5) is matrix A orthogonally diagonalizable ?; that is, there exists an orthogonal matrix P such that $A = PDP^T = PDP^{-1}$ or $P^{-1}AP = P^T AP = D$ with D a diagonal matrix.

4) Solve the diagonalization problem for $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

5) Is the given matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ diagonalizable ?

6) Show that the matrices $A = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -3 & -3 \\ -3 & 0 & -3 \\ -3 & -3 & 0 \end{pmatrix}$

both have 3 as an eigenvalue of multiplicity 2. Show that A is not diagonalizable, but B is diagonalizable.

7) For the given matrix $A = \begin{pmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{pmatrix}$ (Remarks, page 409)

(1) find the eigenvalues

(2) find a set of three mutually orthogonal eigenvectors

8) Problem 31、Problem33 (page 426)

9) Problem 35 (page 426)