Differential Equation	General Solution / Simplifying Method
$g_1(x)h_1(y)dx + g_2(x)h_2(y)dy = 0$	$\int \frac{h_2(y)}{h_1(y)}dy = c - \int \frac{g_1(x)}{g_2(x)}dx$ where $h_1(x)g_2(x) \neq 0$ c is an arbitrary constant to be determined.
$\frac{dy}{dx} = F(\alpha x + \beta y)$	Let $v = \alpha x + \beta y$, $\Rightarrow \frac{dv}{dx} = \alpha + \beta F(v) = G(v)$ $\Rightarrow$ the same form as (1). $\int \frac{1}{G(v)}dv = x + c$ then convert v back to x and y.
$\frac{dy}{dx} = F(\frac{y}{x})$	Let $v = \frac{y}{x}$ $\Rightarrow \frac{dv}{dx} = \frac{1}{x}[F(v) - v] = G(v)$ $\Rightarrow$ the same form as (2).
$\frac{dy}{dx} = F(\frac{x}{y})$	Let $v = \frac{x}{y}$ $\Rightarrow \frac{dv}{dx} = \frac{1}{y}[1 - vF(v)] = G(v)$ $\Rightarrow$ the same form as (2).

$\dfrac{dy}{dx} = F\left(\dfrac{\alpha_1 x + \beta_1 y + \gamma_1}{\alpha_2 x + \beta_2 y + \gamma_2}\right)$	Let $x = X + h$, $y = Y + k$ If $\dfrac{\alpha_1}{\alpha_2} \neq \dfrac{\beta_1}{\beta_2}$, choose h and k such that $\begin{cases} \alpha_1 h + \beta_1 k + \gamma_1 = 0 \\ \alpha_2 h + \beta_2 k + \gamma_2 = 0 \end{cases}$ $\Rightarrow \dfrac{dY}{dX} = F\left(\dfrac{\alpha_1 X + \beta_1 Y}{\alpha_2 X + \beta_2 Y}\right)$ $= F\left(\dfrac{\alpha_1 + \beta_1 Y / X}{\alpha_2 + \beta_2 Y / X}\right) = G\left(\dfrac{Y}{X}\right)$ $\Rightarrow$ the same form as (3) or (4). else if $\dfrac{\alpha_1}{\alpha_2} = \dfrac{\beta_1}{\beta_2} = m$ $\Rightarrow \dfrac{dY}{dX} = F\left(\dfrac{\alpha_1 X + \beta_1 Y}{\alpha_2 X + \beta_2 Y}\right) = G(\alpha X + \beta Y)$ $\Rightarrow$ the same form as (2).
Note: The *above* table is from **eFunda**	

1) $\dfrac{dy}{dx} = e^{3x+2y}$ Ans: $-3e^{-2y} = 2e^{3x} + c$

2) $\dfrac{dy}{dx} = (x+y)^2$ Ans: $x + y = \tan(x+c)$

3) $x\dfrac{dy}{dx} = 4y$ Ans: $y = cx^4$

4) $6x - 2y\dfrac{dy}{dx} = 0$ Ans: $y^2 = 3x^2 + c$

5) $(5x - y + 4) + (x - 5y - 4)\dfrac{dy}{dx} = 0$ Ans: $(y-x)^2(x+y+2)^3 = c$

6) $(2x - 4y + 5)\dfrac{dy}{dx} + (x - 2y + 3) = 0$ Ans: $4x + 8y + \ln\lvert 4x - 8y + 11\rvert = c$