

# 常用微積分公式

## 一、三角恆公式

### 1. 複角公式

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

### 2. 倍角公式

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$$

### 3. 半角公式

$$\sin^2(x) = \frac{1 - \cos 2x}{2}, \sin^3(x) = \frac{3 \sin x - \sin 3x}{4}$$

$$\cos^2(x) = \frac{1 + \cos 2x}{2}, \cos^3(x) = \frac{3 \cos x + \cos 3x}{4}$$

### 4. 積化和差

$$\sin x \cos y = \frac{1}{2} [\sin(x - y) + \sin(x + y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

## 二、雙曲線恆等式

### 1. 倍角公式

$$\cosh(2x) = 2 \cosh^2(x) - 1$$

$$\sinh(2x) = 2 \cosh x \sinh x$$

### 2. 恒等式

$$\cosh^2 x - \sinh^2 x = 1, 1 - \tanh^2 x = \sec h^2 x$$

$$\coth^2 x - 1 = \csc h^2 x$$

## 三、重要函數微分

### 1. 三角函數

$$(\tan x)' = \sec^2 x, \quad (\cot x)' = -\csc^2 x$$

$$(\sec x)' = \sec x \tan x, (\csc x)' = -\csc x \cot x$$

## 2. 反三角函數

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \frac{1}{1+x^2}$$

## 3. 反雙曲線函數

$$(\sinh^{-1} x)' = \frac{1}{\sqrt{1+x^2}}, \quad (\tanh^{-1} x)' = \frac{1}{1-x^2}$$

## 四、重要函數積分

$$\int \sec^2 x dx = \tan x$$

$$\int \csc^2 x dx = -\cot x$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \csc x \cot x dx = -\csc x$$

$$\int \sec x dx = \ln |\sec x + \tan x|$$

$$\int \csc x dx = \ln |\csc x - \cot x|$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + b \sin bx]$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} [a \sin bx - b \cos bx]$$

$$\int e^{ax^2} dx = \frac{1}{2a} e^{ax^2}$$

## 五、重要定積分

$$\int_0^\infty e^{-ax^2} \cos bx dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-\frac{b^2}{4a}}$$

$$\int_{-\infty}^\infty e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}, \int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$