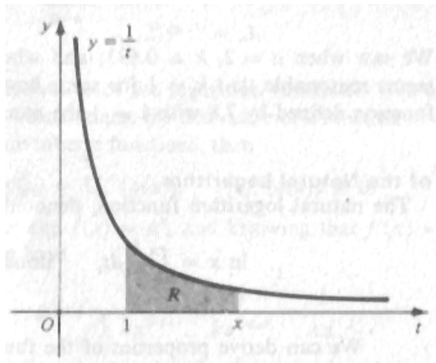
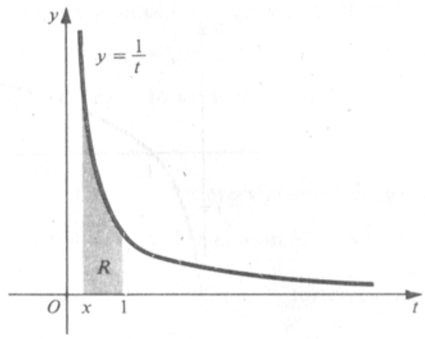
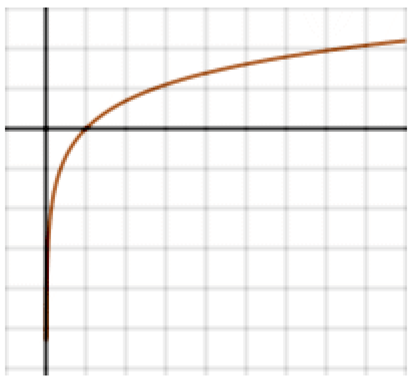
Definition of the natural logarithm (自然對數的定義)

$$\ln x = \int_1^x \frac{1}{t}\,dt \qquad \text{domain } \ln = (0,\infty)$$





幾何上而言，若 x>1，則 ln(x) 爲區域 R 的面積；若 0<x<1，則 - ln(x) 爲區域 R 的面積。



ln(x)

$$\ln x = \log_e x,\quad e \cong 2.71828$$

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$$D_x \ln x = \frac{1}{x}$$

$$D_x^2 \ln x = -\frac{1}{x^2}$$

$$\ln(ab) = \ln a + \ln b$$

$$\ln\frac{a}{b} = \ln a - \ln b$$

$$\ln a^r = r\ln a$$

$$\ln e = 1$$
$$\log_a a = 1$$
$$\log_a xy = \log_a x + \log_a y$$
$$\log_a x^p = p \log_a x$$
$$\frac{d \ln x}{dx} = \frac{1}{x}$$
$$\frac{d^n \ln x}{dx^n} = (-1)^{n-1} (n-1)! \, x^{-n}$$

$$\ln e^x = x$$
$$\log_a a^x = x$$
$$\log_a (x/y) = \log_a x - \log_a y$$
$$\log_a x = \frac{\log_b x}{\log_b a}$$
$$\int \frac{1}{x} dx = \ln x$$
$$\int \ln x dx = x \ln x - x$$

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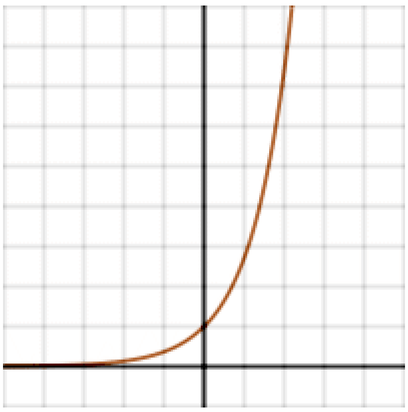
<u>The **natural exponential function (自然指數函數)**</u>

$\exp x = e^x$ domain $\exp = (-\infty, \infty)$



e^x

As a function of the *real* variable *x*, the graph of e^x is always positive (above the *x* axis) and increasing (viewed left-to-right).

$$e^x e^y = e^{x+y}$$
$$\left(e^x\right)^p = e^{px}$$
$$\frac{de^x}{dx} = e^x$$
$$\frac{de^{ax}}{dx} = ae^{ax}$$
$$\frac{d^n e^{ax}}{dx^n} = a^n e^{ax}$$

$$e^x / e^y = e^{x-y}$$
$$\sqrt[p]{e^x} = e^{x/p}$$
$$\int e^x dx = e^x$$
$$\int e^{ax} dx = \frac{e^{ax}}{a}$$
$$\int \cdots \int e^{ax} \underbrace{dx \ldots dx}_{n} = \frac{e^{ax}}{a^n}$$

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Note:

Set $\ln(x) = y$ ➔ $e^{\ln(x)} = x = e^{y}$ ➔ x > 0 (e^{y} is always positive)

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EX: The natural logarithm can be integrated using integration by parts

$$\int \ln(x)\,dx = x\ln(x) - x + C$$

各位同學，請試著導出上式!!!