Engineering Mathematics I---Quiz-1s

1) **Solve** $\dfrac{dy}{dx} = \dfrac{2y^2 + y - 3}{3y + 2}$ (**separable DE**)

ANS $\rightarrow \displaystyle\int \dfrac{3y + 2}{2y^2 + y - 3}\,dy = \int dx$

$\rightarrow \displaystyle\int \left(\dfrac{1}{2y + 3} + \dfrac{1}{y - 1} \right) dy = x + c$

$\rightarrow \dfrac{1}{2}\ln(2y + 3) + \ln(y - 1) = x + c$

$\rightarrow \ln\!\left((2y + 3)^{\frac{1}{2}}(y - 1) \right) = x + c$

$\rightarrow (2y + 3)^{\frac{1}{2}}(y - 1) = ce^{x}$

2) **Solve** $(5x - y + 4) + (x - 5y - 4)\dfrac{dy}{dx} = 0$ (**separable DE**)

ANS $\dfrac{dy}{dx} = \dfrac{-5x + y - 4}{x - 5y - 4}$

$\begin{cases} -5x + y - 4 = 0 \\ x - 5y - 4 = 0 \end{cases} \rightarrow \begin{cases} x = -1 \\ y = -1 \end{cases}$

Let $u = x + 1$, $v = y + 1$
$\rightarrow du = dx$, $dv = dy$

$\rightarrow \dfrac{dy}{dx} = \dfrac{dv}{du} = \dfrac{-5u + v}{u - 5v} = \dfrac{-5 + \dfrac{v}{u}}{1 - 5\dfrac{v}{u}}$

Let $w = \dfrac{v}{u} \rightarrow v = wu$ $\rightarrow \dfrac{dv}{du} = \dfrac{dw}{du}u + w$

$\rightarrow \dfrac{-5 + w}{1 - 5w} = u\dfrac{dw}{du} + w$

$\rightarrow u\dfrac{dw}{du} = \dfrac{-5 + w}{1 - 5w} - \dfrac{w - 5w^2}{1 - 5w} = \dfrac{5w^2 - 5}{1 - 5w}$

$\rightarrow \displaystyle\int \dfrac{1 - 5w}{5w^2 - 5}\,dw = \int \dfrac{1}{u}\,du$ $\rightarrow \displaystyle\int \dfrac{-3}{w + 1} + \dfrac{-2}{w - 1}\,dw = \int \dfrac{5}{u}\,du$

$\rightarrow \ln(w + 1)^{-3} + \ln(w - 1)^{-2} = \ln u^{5} + c$

$\rightarrow \ln\!\left((w + 1)^{-3}(w - 1)^{-2} \right) = \ln u^{5} + c$

$\rightarrow \left(\dfrac{y + 1}{x + 1} + 1 \right)^{-3}\left(\dfrac{y + 1}{x + 1} - 1 \right)^{-2} = c(x + 1)^{5}$

$\rightarrow (y - x)^{2}(x + y + 2)^{3} = c$

3) **To determine the degree, order and linear of the DE.**

(1) $(1-x)y'' - 4xy' + 5y = \cos x$

(2) $\dfrac{d^2y}{dx^2} = \sqrt{1+(\dfrac{dy}{dx})^2}$

ANS (1)2   1

(2)2   2