

Engineering Mathematics I---Quiz-2s

1) **Solve** $t^2 \frac{dy}{dt} + y^2 = ty$

ANS $\rightarrow \frac{dy}{dt} + \frac{1}{t^2} y^2 = \frac{1}{t} y \rightarrow \frac{dy}{dt} - \frac{1}{t} y = -\frac{1}{t^2} y^2$ Bernoulli's Eq.

同除 $y^2 \rightarrow \frac{1}{y^2} \frac{dy}{dt} - \frac{1}{t} \frac{1}{y} = -\frac{1}{t^2}$

令 $z = \frac{1}{y^{2-1}} \rightarrow \frac{dz}{dt} = -\frac{1}{y^2} \frac{dy}{dt}$ 代入 ODE

$\rightarrow \frac{dz}{dt} + \frac{1}{t} z = \frac{1}{t^2}$ 一階線性 ODE

$$I(t) = e^{\int \frac{1}{t} dt} = t$$

$\rightarrow t \frac{dz}{dt} + z = \frac{1}{t} \rightarrow \frac{d}{dt} [tz] = \frac{1}{t}$

$\rightarrow tz = \ln t + c \rightarrow \frac{t}{y} = \ln t + c \rightarrow y^{-1} = \frac{1}{t} \ln t + \frac{c}{t}$

2) **Solve** $2x \frac{dy}{dx} - y^2 + 2y + 8 = 0$

[Hint: One solution is of the form b (b is a constant)]

ANS $\frac{dy}{dx} = \frac{1}{2x} y^2 - \frac{1}{x} y - \frac{4}{x}$ Ricatti's Eq.

求一特解

令 $y = b$ 代入 $\rightarrow -b^2 + 2b + 8 = 0 \rightarrow b = 4 \text{ or } 2$

令 $y = 4 + \frac{1}{z}$ 因變數變換

$\rightarrow y' = -\frac{z'}{z^2} = \frac{1}{2x} (4 + \frac{1}{z})^2 - \frac{1}{x} (4 + \frac{1}{z}) - \frac{4}{x} \rightarrow y' = \frac{1}{2x} (16 + \frac{8}{z} + \frac{1}{z^2}) - \frac{1}{x} (4 + \frac{1}{z}) - \frac{4}{x}$

$\rightarrow 2xz' + 6z + 1 = 0 \rightarrow z' + \frac{3}{x} z = -\frac{1}{2x}$

$$I(x) = e^{\int \frac{3}{x} dx} = x^3$$

$\rightarrow x^3 z = \int x^3 (-\frac{1}{2x}) dx + c \rightarrow x^3 z = -\frac{1}{2} \int x^2 dx + c$

$\rightarrow x^3 z = -\frac{1}{6} x^3 + c \rightarrow z = -\frac{1}{6} + \frac{c}{x^3} \rightarrow y(x) = 4 + \frac{1}{-\frac{1}{6} + \frac{c}{x^3}}$