

Engineering Mathematics I---Quiz-3

1) $y'' - 3y' + 2y = 0$

ANS 令 $y = e^{mx}$ 代入 ODE

$$\rightarrow m^2 e^{mx} - 3m e^{mx} + 2e^{mx} = 0$$

$$\rightarrow (m^2 - 3m + 2)e^{mx} = 0$$

$$\rightarrow m^2 - 3m + 2 = 0 \rightarrow m = 1, 2$$

$$\rightarrow y = c_1 e^x + c_2 e^{2x}$$

2) $(1-x^2)y'' - 2xy' + 2y = 0, -1 < x < 1, (y_1 = x)$

ANS 令 $y = xu$ 代入 $\rightarrow y' = u + xu', y'' = 2u' + xu''$

$$\rightarrow (1-x^2)(2u' + xu'') - 2x(u + xu') + 2(xu) = 0$$

$$\rightarrow (x-x^3)u'' + (2-2x^2-2x^2)u' + (-2x+2x)u = 0$$

令 $u' = p$ 代入

$$\rightarrow \frac{dp}{dx} + \frac{2(1-2x^2)}{x(1-x^2)} p = 0 \text{ 可分離變數}$$

$$\rightarrow \frac{dp}{p} + \frac{2(2x^2-1)}{x(x-1)(x+1)} dx = 0$$

$$\rightarrow \frac{dp}{p} + \left(\frac{2}{x} + \frac{1}{x-1} + \frac{1}{x+1} \right) dx = 0$$

$$\rightarrow \ln p + 2 \ln x + \ln(x-1) + \ln(x+1) = c' \text{ 積分}$$

$$\rightarrow p = \frac{c_1}{x^2(x^2-1)} \text{ 取指數}$$

$$\rightarrow u' = \frac{c_1}{x^2(x^2-1)} = \frac{c_1}{x^2(x-1)(x+1)}$$

$$\rightarrow u = c_1 \int \frac{1}{x^2(x^2-1)} dx + c_2 \rightarrow u = c_1 \int \frac{-1}{x^2} + \frac{1}{2} \frac{1}{x-1} + \frac{-1}{2} \frac{1}{x+1} dx + c_2$$

$$\rightarrow u = c_1 \left[\frac{1}{x} + \frac{1}{2} \ln \frac{x-1}{x+1} \right] + c_2$$

$$\rightarrow y = x \left[c_1 \left(\frac{1}{x} + \frac{1}{2} \ln \frac{x-1}{x+1} \right) + c_2 \right] \rightarrow y = c_2 \left(1 + \frac{x}{2} \ln \frac{x-1}{x+1} \right) + c_1 x$$

$$\rightarrow c_1 y_1 + c_2 y_2$$