

Engineering Mathematics I---Quiz-4s

1) Find the general solution of the given second-order differential equation.

$$3y'' + 2y' + y = 0$$

ANS From $3m^2 + 2m + 1 = 0$ we obtain $m = -1/3 \pm \sqrt{2}i/3$ so that

$$y = e^{-x/3} (c_1 \cos \sqrt{2}x/3 + c_2 \sin \sqrt{2}x/3).$$

2) Find the differential equation of the general second-order solution.

$$y = e^x (\cos 2x + 2 \sin 2x)$$

ANS $y'' - 2y' + 5y = 0$

3) Solve the given initial-value problem

$$y''' - 2y'' + y' = 2 - 24e^x + 40e^{5x}; \quad y(0) = \frac{1}{2}, \quad y'(0) = \frac{5}{2}, \quad y''(0) = -\frac{9}{2}$$

ANS We have $y_c = c_1 + c_2e^x + c_3xe^x$ and we assume $y_p = Ax + Bx^2e^x + Ce^{5x}$.

Substituting into the differential equation we find $A = 2$, $B = -12$, and $C = \frac{1}{2}$. Thus $y = c_1 + c_2e^x + c_3xe^x + 2x - 12x^2e^x + \frac{1}{2}e^{5x}$. From the initial conditions we obtain $c_1 = 11$, $c_2 = -11$, and $c_3 = 9$, so

$$y = 11 - 11e^x + 9xe^x + 2x - 12x^2e^x + \frac{1}{2}e^{5x}.$$