

Engineering Mathematics I---Quiz-5s

1) **Solve** $y'' + y = \cos^2 x$

ANS 1. $\text{令 } y_H = e^{mx}$ 代入 $y'' + y = 0$

$$\rightarrow m^2 + 1 = 0$$

$$\rightarrow m = \pm i$$

$$\rightarrow y_H = c_1 \cos x + c_2 \sin x$$

2. $\text{令 } y_p = \phi_1(x) \cos x + \phi_2(x) \sin x$

$$w = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1 \quad w_1 = \begin{vmatrix} 0 & \sin x \\ \cos^2 x & \cos x \end{vmatrix} = -\sin x \cos^2 x$$

$$w_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \cos^2 x \end{vmatrix} = \cos^3 x$$

$$\rightarrow \phi_1'(x) = \frac{w_1}{w} = -\sin x \cos^2 x \quad \rightarrow \phi_1(x) = \frac{1}{3} \cos^3 x$$

$$\rightarrow \phi_2'(x) = \frac{w_2}{w} = \cos^3 x = \cos x(1 - \sin^2 x) \quad \rightarrow \phi_2(x) = \sin x - \frac{1}{3} \sin^3 x$$

$$\rightarrow y_p = \left(\frac{1}{3} \cos^3 x\right) \cos x + \left(\sin x - \frac{1}{3} \sin^3 x\right) \sin x \quad \rightarrow y_p = \frac{1}{3} \cos^4 x + \sin^2 x - \frac{1}{3} \sin^4 x$$

$$\rightarrow y_p = \frac{1}{3} (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) + \sin^2 x$$

$$\rightarrow y_p = \frac{1}{3} \cos^2 x + \frac{2}{3} \sin^2 x$$

$$3. y = y_H + y_p = c_1 \cos x + c_2 \sin x + \frac{1}{3} \cos^2 x + \frac{2}{3} \sin^2 x$$

2) **Solve** $x^2 y'' - 3xy' + 3y = 2x^4 e^x$

ANS 1. $\text{令 } y_H = x^m \quad y' = mx^{m-1} \quad y'' = m(m-1)x^{m-2}$

$$\rightarrow m(m-1)x^m - 3mx^m + 3x^m = 0 \quad \rightarrow m^2 - 4m - 3 = 0$$

$$\rightarrow (m-1)(m-3) = 0 \quad \rightarrow y_H = c_1 x + c_2 x^3$$

2. $\text{令 } y_p = u_1 y_1 + u_2 y_2 = u_1 x + u_2 x^3$

$$w = \begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix} = 2x^3 \quad w_1 = \begin{vmatrix} 0 & x^3 \\ 2x^2 e^x & 3x^2 \end{vmatrix} = -2x^5 e^x \quad w_2 = \begin{vmatrix} x & 0 \\ 1 & 2x^2 e^x \end{vmatrix} = 2x^3 e^x$$

$$\rightarrow u_1' = -\frac{2x^5 e^x}{2x^3} = -x^2 e^x \quad \rightarrow u_2' = \frac{2x^3 e^x}{2x^3} = e^x$$

$$\rightarrow u_1 = -x^2 e^x + 2xe^x - 2e^x \quad \rightarrow u_2 = e^x \quad \rightarrow y_p = 2x^2 e^x - 2xe^x$$

$$3. y = y_H + y_p = c_1 x + c_2 x^3 + 2x^2 e^x - 2xe^x$$