Engineering Mathematics I---Quiz-6s

1) **Given a second-order differential equation** $y'' + 5y' + 4y = 0$ **.**

Use two method to solve the problem.

ANS 1. Let $y = e^{mx}$

$$\to m^2 + 5m + 4 = 0 \quad \to m = -1\ ,\ -4 \quad \to y = c_1 e^{-x} + c_2 e^{-4x}$$

2. Let $y = \sum_{n=0}^{\infty} anx^n$, $y' = \sum_{n=1}^{\infty} na_n x^{n-1}$, $y'' = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$

$$\to y'' + 5y' + 4y = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + 5\sum_{n=1}^{\infty} na_n x^{n-1} + 4\sum_{n=0}^{\infty} a_n x^n$$

Let $n-2=k$ $\quad n-1=k$ $\quad n=k$

$$\to y'' + 5y' + 4y = \sum_{k=0}^{\infty} (k+2)(k+1)a_{k+2}x^k + 5\sum_{k=0}^{\infty} (k+1)a_{k+1}x^k + 4\sum_{k=0}^{\infty} a_k x^k$$

$$\to y'' + 5y' + 4y = \sum_{k=0}^{\infty} [(k+2)(k+1)a_{k+2} + 5(k+1)a_{k+1} + 4a_k]x^k = 0$$

$$\to (k+2)(k+1)a_{k+2} + 5(k+1)a_{k+1} + 4a_k = 0$$

$$\to a_{k+2} = \frac{-5(k+1)a_{k+1} - 4a_k}{(k+2)(k+1)},\ k = 0,1,2,\cdots$$

K=0 $\to a_2 = \dfrac{-5a_1 - 4a_0}{2} = -\dfrac{4}{2}a_0 - \dfrac{5}{2}a_1$

K=1 $\to a_3 = \dfrac{-10a_2 - 4a_1}{6} = \dfrac{10}{3}a_0 + \dfrac{7}{2}a_1$

K=2 $\to a_4 = \dfrac{-15a_3 - 4a_2}{12} = -\dfrac{7}{2}a_0 - \dfrac{85}{24}a_1$

K=3 $\to a_5 = \dfrac{-20a_4 - 4a_3}{20} = \dfrac{17}{6}a_0 + \dfrac{341}{120}a_1$

K=4 $\to a_6 = \dfrac{-25a_5 - 4a_4}{30} = -\dfrac{341}{180}a_0 - \dfrac{91}{48}a_1$

K=5 $\to a_7 = \dfrac{-30a_6 - 4a_5}{42} = \dfrac{13}{12}a_0 + \dfrac{5461}{5040}a_1$

⋮ ⋮

$$\to y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

$$\to y = a_0[1 + (-\frac{4}{2})x^2 + \frac{10}{3}x^3 + (\frac{-7}{2})x^4 + \frac{17}{6}x^5 + (\frac{-341}{180})x^6 + \frac{13}{12}x^7 + \ldots]$$

$$+ a_1[x + (\frac{-5}{2})x^2 + \frac{7}{2}x^3 + (\frac{-85}{24})x^4 + \frac{341}{120}x^5 + (\frac{-91}{48})x^6 + \frac{5461}{5040}x^7 + \ldots]$$