

Engineering Mathematics I---Quiz-7s

- 1) Write out the standard form of Bessel's equation .
- 2) Write out the standard form of Legendre's equation .
- 3) Explain what the Legendre polynomials $P_n(x)$ is .
- 4) Solve the Bessel's equation $x^2y'' + xy' + (x^2 - \frac{25}{4})y = 0$.

ANS 1) $x^2y'' + xy' + (x^2 - \mu^2)y = 0$

2) $(1-x^2)y'' - 2xy' + \mu(\mu+1)y = 0$

- 3) The specific n th-degree polynomial solutions are called Legendre polynomials and are denoted by $P_n(x)$.

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$$

4) $\mu = \pm \frac{5}{2}$

$$J_{5/2}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(n + \frac{5}{2} + 1)} \left(\frac{x}{2}\right)^{2n + \frac{5}{2}}$$

$$J_{-5/2}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(n - \frac{5}{2} + 1)} \left(\frac{x}{2}\right)^{2n - \frac{5}{2}}$$

$$y = c_1 J_{5/2}(x) + c_2 J_{-5/2}(x)$$