1, Using cofactor expansions, combined with elementary row and column operations when this is useful, to evaluate the determinat of the matrix (P326 Problem 5)

$$\begin{pmatrix} -5 & 0 & 1 & 6 \\ 2 & -1 & 3 & 7 \\ 4 & 4 & -5 & -8 \\ 1 & -1 & 6 & 2 \end{pmatrix}$$

Sol :

對第一列做降階 :

$$原式 = -5\begin{pmatrix} -1 & 3 & 7 \\ 4 & -5 & -8 \\ -1 & 6 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 7 \\ 4 & 4 & -8 \\ 1 & -1 & 2 \end{pmatrix} - 6\begin{pmatrix} 2 & -1 & 3 \\ 4 & 4 & -5 \\ 1 & -1 & 6 \end{pmatrix}$$

$= -5[(10 + 24 + 168) - (35 + 24 + 48)] + [(16 + 8 - 28) - (28 + 16 - 8)] - 6[(48 - 12 + 5) - (12 + 10 - 24)]$

$= -5 * 95 - 40 - 6 * 43$

$= -773$

2, Produce a matrix that diagonalizes the given matrix or show that this matrix is not diagonalizable (P353 Problem 5 & Problem 7)

(a) $\begin{pmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{pmatrix}$

(b) $\begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

Sol :

(a)

$[A] = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{pmatrix}$

$\det[(A - \lambda I)] = 0$

$\det\left[\begin{pmatrix} 5-\lambda & 0 & 0 \\ 1 & -\lambda & 3 \\ 0 & 0 & -2-\lambda \end{pmatrix}\right] = 0$

$(5 - \lambda)(-\lambda)(-2 - \lambda) = 0$

$\lambda = 5, -2, 0$

$\lambda = 5$ 時,

$[A - 5I]\{v_1\} = \{0\}$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & -5 & 3 \\ 0 & 0 & -7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 - 5x_2 + 3x_3 = 0$

令$x_3 = 0$, $x_2 = 1$

$\{v_1\} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$

$\lambda = -2$ 時,

$(A + 2I)\{v_2\} = \{0\}$

$$\begin{pmatrix} 7 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 + 2x_2 + 3x_3 = 0$

令$x_3 = 0$, $x_2 = 1$

$\{v_2\} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$

$\lambda = 0$ 時,

$(A - 0\,I)\{v_3\} = \{0\}$

$\begin{pmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x_1 + 3\,x_3 = 0$

令$x_3 = 0$, $x_2 = 1$

$\{v_3\} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

令$[P] = [\{v_1\}, \{v_2\}, \{v_3\}] = \begin{pmatrix} 5 & 0 & 0 \\ 1 & -3 & 1 \\ 0 & 2 & 0 \end{pmatrix}$

則$[P]^{-1}[A][P] = \begin{pmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(b)

$[A] = \begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

$\det[(A - \lambda I)] = 0$

$\det\left[\begin{pmatrix} -2-\lambda & 0 & 1 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{pmatrix}\right] = 0$

$(-2-\lambda)^2\,(1-\lambda) = 0$

$\lambda = 1, -2$ (重根)

$\lambda = 1$

$(A - I)\{v_1\} = \{0\}$

$\begin{pmatrix} -3 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$-3\,x_1 + x_3 = 0$

令$x_2 = 1$, $x_1 = 0$

$\{v_3\} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda = -2$

$(A - I)\{v_2\} = \{0\}$

$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x_1 = 0$

令$x_2 = 1$, $x_3 = 1$

$\{v_2\} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

只能找出兩特徵向量，所以矩陣不可對角化

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