1, $y = e^{kx}$

sol : $y' = k * e^{kx}$

$\ln y = kx \ln e = kx$

$k = \frac{\ln y}{x}$

$y' = \frac{y \ln y}{x} = f(x, y)$

$R : y' = \frac{-1}{f(x, y)} = \frac{-x}{y \ln y}$

$\frac{dy}{dx} = \frac{-x}{y \ln y}$

$y \ln y \, dy = -x dx$

$\int y \ln y \, dy = -\int x \, dx$

$\Rightarrow$ By part $\int y \ln y \, dy$

$\frac{1}{2} y^2 \ln y - \frac{1}{4} y^2 + c1 = \frac{-1}{2} x^2 + c2$

$2 y^2 \ln y - y^2 + 2 x^2 = c$

$y^2 [\ln (y^2) - 1] + 2 x^2 = c$

2, (1), $y' = 8 x^3 - 3 y$

sol :

$y' + 3 y = 8 x^3$

$I(x) = e^{\int 3 dx} = e^{3x}$

$e^{3x} \, y = 8 \int e^{3x} x^3 \, dx + c$

$\Rightarrow$ By part $\int e^{3x} x^3 \, dx$

$e^{3x} \, y = \frac{8}{3} e^{3x} \left[x^3 - x^2 + \frac{2}{3} - \frac{2}{9}\right]$

$\therefore y = \frac{8}{3} x^3 - \frac{8}{3} x^2 + \frac{16}{9} x - \frac{16}{27} + c e^{-3x}$

(2), $2 y - 7 x - 2 (y - x) y' = 0$

sol :

$2 y - 7 x - 2 (y - x) \frac{dy}{dx} = 0$

$(2 y - 7 x) dx - 2 (y - x) dy = 0$

找$\phi$

$\Rightarrow \phi = \int (2 y - 7 x) dx + g(y)$

$= 2 xy - \frac{7}{2} + g(y)$

$\phi = \int (-2 y + 2 x) dy + f(x)$

$= 2 xy - y^2 + f(x)$

$\phi(x, y) = \frac{-7}{2} x^2 + 2 xy - y^2 = c$

(3), $(x^2 - 4) y' = y + 3$

sol :

$(x^2 - 4) \frac{dy}{dx} = y + 3$

$\int \frac{1}{x^2 - 4} dx = \int \frac{1}{y + 3} dy$

$$\int \frac{1}{(x+2)\ (x-2)}\ dx = \int \frac{1}{y+3}\ dy$$

$$\ln\ |\ y+3\ |\ = \frac{1}{4}\ \ln \left| x+2 \right| - \frac{1}{4}\ \ln \left| x-2 \right| + c$$

$$y+3 = \left(\frac{x+2}{x-2}\right)^{\frac{1}{4}} * e^{c} - 3$$

(4), $xy' + y = 2\,y^{\frac{3}{2}}$

sol :

$$y' + \frac{y}{x} = \frac{2}{x}\ y^{\frac{3}{2}}$$

let $v = y^{1-\frac{3}{2}} = y^{\frac{-1}{2}}$

$$y = v^{-2}$$

$$y' = -2\,v^{-3}\,v'$$

$$-2\,v^{-3}\,v' + \frac{v^{-2}}{x} = \frac{2}{x}\ v^{-3}$$

$$v' - \frac{1}{2\,x}\ v = -\frac{1}{x}$$

$$I = e^{\int \frac{-1}{2x}\,dx} = x^{\frac{-1}{2}}$$

$$Iv = \int -x^{\frac{-1}{2}}\ x^{-1}\,dx \quad = 2\,x^{\frac{-1}{2}} + c$$

$$v = 2 + cx^{\frac{1}{2}}$$

$$y = \left(2 + cx^{\frac{1}{2}}\right)^{-2}$$

(5), $6\,x - 2\,yy' = 0$

sol :

$$yy' = 3\,x$$

$$y\,dy = 3\,x\,dx$$

$$y^2 = 3\,x^2 + c$$

(6), $y' = \dfrac{4\,y}{4\,x - y}$

sol :

$$y' = \frac{4\ (\frac{y}{x})}{4 - (\frac{y}{x})}$$

let $u = \dfrac{y}{x}$

$$y' = u + x\,u'$$

$$u + x\,u' = \frac{4\,u}{4 - u}$$

$$x\ \frac{du}{dx} = \frac{u^2}{4 - u}$$

$$\frac{4 - u}{u^2}\ du = \frac{1}{x}\ dx$$

$$\ln[x] = -4\,u^{-1} - \ln[u] + c$$

$$x = e^{-4\,u^{-1} - \ln[u] + c}$$

$$x = \frac{c e^{-4\,u^{-1}}}{u}$$

$$y = c e^{\frac{-4\,x}{y}}$$

$$\ln[y] = c - \frac{4\,x}{y}$$

$$y\ \ln[y] + 4\,x = cx$$

(7), $y' = x y^2 + (1 - 2x) y + (x + 1)$

sol :

particular solution $y = 1$

let $y = 1 + \frac{1}{z} \qquad y' = -\frac{z'}{z^2}$

$$-\frac{z'}{z^2} = x\left(1 + \frac{1}{z}\right)^2 + (1 - 2x)\left(1 + \frac{1}{z}\right) + (x + 1)$$

$$-z' = x(z^2 + 2z + 1) + (z^2 - 2xz^2 + z - 2xz) + z^2(x - 1)$$

$$z' + z = -x$$

$$I = e^{\int 1\, dx} = e^x$$

$$Iz = \int -x e^x\, dx = x e^x - e^x + c$$

$$z = -x + 1 + c e^{-x}$$

$$y = 1 + \frac{1}{1 - x + c e^{-x}}$$