1, $y^2 - 4x^2 + e^{xy} = c$, $8x - ye^{xy} - (2y + xe^{xy})\, y' = 0$

Sol：

$$\frac{d}{dx}\left(y^2 - 4x^2 + e^{xy} = c\right)$$

$\Longrightarrow 2yy' - 8x + e^{xy}(y + xy') = 0$

$\Longrightarrow 8x - ye^{xy} - (2y + xe^{xy})\, y' = 0$

所以$y^2 - 4x^2 + e^{xy} = c$爲$8x - ye^{xy} - (2y + xe^{xy})\, y' = 0$的解

2, $(x^2 - 4)\, y' = y + 3$

Sol：

$$(x^2 - 4)\frac{dy}{dx} = y + 3$$

$$\int \frac{dy}{y+3} = \int \frac{dx}{(x^2 - 4)} = \int \left(\frac{\frac{-1}{4}}{(x+2)} + \frac{\frac{1}{4}}{(x-2)}\right) dx$$

$$\ln|y+3| = \frac{-1}{4}\left(\ln|x+2| - \ln|x-2|\right) + c$$

$$\ln|y+3| = \ln\left|\frac{x-2}{x+2}\right|^{\frac{1}{4}} + c$$

$$y = e^c\left(\frac{x-2}{x+2}\right)^{\frac{1}{4}} - 3$$

$$y = k\left(\frac{x-2}{x+2}\right)^{\frac{1}{4}} - 3$$

3, $y - xy' = 0$

Sol：

(a)

$M = y$, $N = -x$

$$\frac{\partial M}{\partial y} = 1 \neq \frac{\partial N}{\partial x} = -1$$

故不爲正合

(b)

$\mu(x)\, y - \mu(x)\, x\, y' = 0$

$$\frac{\partial M}{\partial y} = \mu\, , \quad \frac{\partial N}{\partial x} = -\mu' x - \mu$$

$\mu = -x\mu' - \mu$

$$2\mu = -x\frac{d\mu}{dx}$$

$$\int \frac{-1}{2\mu}\, d\mu = \int \frac{1}{x}\, dx$$

$$\frac{-1}{2}\ln|\mu| = \ln|x|$$

$\mu = x^{-2}$

(c)

integrate factor $\eta(x, y) = x^a y^b$

$x^a y^b (y) - x^a y^b (x y') = 0$

$x^a y^{b+1} - x^{a+1} y^b y' = 0$

$M = x^a y^{b+1}$, $N = -x^{a+1} y^b$

$$\frac{\partial M}{\partial y} = (b+1)\, x^a y^b\, , \quad \frac{\partial N}{\partial x} = -(a+1)\, x^a y^b$$

$(b+1)\, x^a y^b = -(a+1)\, x^a y^b$

$a + b = -2$

4, $y' = \frac{3x + y - 1}{6x + 2y - 3}$

Sol :

$$y' = \frac{\frac{3x+y-1}{3}}{\frac{6x+2y-3}{3}}$$

$$\text{set } u = \frac{3x + y}{3} \implies y' = 3u' - 3$$

$$3u' - 3 = \frac{u - \frac{1}{3}}{2u - 1}$$

$$9u' = \frac{3u - 1}{2u - 1} + 9$$

$$u' = \frac{21u - 10}{18u - 9}$$

$$\frac{du}{dx} = \frac{21u - 10}{18u - 9}$$

$$\int \frac{18u - 9}{21u - 10}\, du = \int dx$$

$$\int \left(\frac{6}{7} - \frac{\frac{3}{7}}{21u - 10} \right) du = x + c$$

$$\frac{6}{7} u - \frac{1}{49} \ln \left| 21u - 10 \right| = x + c$$

$$-7x + 14y - \ln|21x + 7y - 10| = c_1$$

5, $x y' = -y + x^2 y^2$

Sol :

$$y' + \frac{1}{x} y = x y^2$$

$$\text{set } u = y^{-1} \implies y' = -u^{-2} u'$$

$$-u^{-2} u' + \frac{1}{x} u^{-1} = x u^{-2}$$

$$u' - \frac{1}{x} u = -x$$

integrate factor

$$I = e^{\int \frac{-1}{x} dx} = x^{-1}$$

$$Iu = \int (x^{-1} * -x)\, dx = -x + c$$

$$u = -x^2 + cx$$

$$y = \frac{1}{-x^2 + cx}$$

6, $y' = -\frac{y^2}{x} + \frac{2y}{x}$

Sol :

$$y' - \frac{2y}{x} = -\frac{y^2}{x}$$

$$\text{set } u = y^{-1} \implies y' = -u^{-2} u'$$

$$-u^{-2} u' - \frac{2}{x} u^{-1} = -\frac{1}{x} u^{-2}$$

$$u' + \frac{2}{x} u = \frac{1}{x}$$

integrate factor

$$I = e^{\int \frac{2}{x} dx} = x^2$$

$$Iu = \int \left(x^2 * \frac{1}{x} \right) dx = \frac{1}{2} x^2 + c$$

$$u = \frac{1}{2} + cx^{-2}$$

$$y = \frac{1}{\frac{1}{2} + cx^{-2}} = \frac{2}{2 + kx^{-2}}$$

7, $\frac{1}{2} x^2 + y^2 = c$

Sol :

$$y^2 = \frac{-1}{2} x^2 + c$$

$$2 y y_1' = -x$$

$$y_1' = \frac{-x}{2 y}$$

$$y_1' * y_2' = -1$$

$$y_2' = \frac{2 y}{x}$$

$$\frac{dy}{dx} = \frac{2 y}{x}$$

$$\ln | y | = \ln | x^2 | + c$$

$$y = A x^2$$