Mid-term Exam I

1) Verify by implicit differentiation that the given equation implicitly defines a solution of the differential equation (5 scores)

$$y^2 - 4x^2 + e^{xy} = C \; ; \quad 8x - ye^{xy} - (2y + xe^{xy})y' = 0$$

2) Solve $(x^2 - 4)y' = y + 3$ (10 scores)

3) Consider $y - xy' = 0$ (25 scores)

  (a)Show that this equation is not exact on any rectangle (5 scores)

  (b)Find an integrating factor $\mu(x)$ that is a function of x alone (10 scores)

  (c)Show that there is also an integrating factor $\eta(x, y) = x^a y^b$ for some constants a and b. Find all such integrating factor (10 scores)

4) Solve $y' = \dfrac{3x + y - 1}{6x + 2y - 3}$ (15 scores)

5) Solve $xy' = -y + x^2 y^2$ (15 scores)

6) Solve $y' = -\dfrac{y^2}{x} + \dfrac{2y}{x}$ (15 scores)

7) Given a family of curves $\dfrac{1}{2}x^2 + y^2 = C$, find the family of orthogonal trajectories of the given family of curves (15 scores)