

國立台灣海洋大學 2005 河工系工程數學(二) 第一次作業題目

13.2

In each of Problems 1 through 12, find the Fourier series of the function on the interval.

$$5. f(x) = \begin{cases} -4 & \text{for } -\pi \leq x \leq 0 \\ 4 & \text{for } 0 < x \leq \pi \end{cases}$$

$$9. f(x) = \begin{cases} 1 & \text{for } -\pi \leq x < 0 \\ 2 & \text{for } 0 \leq x \leq \pi \end{cases}$$

14. Prove that $\int_{-L}^L f(x)dx = 0$ if f is odd on $[-L, L]$.

15. Prove that $\int_{-L}^L f(x)dx = 2\int_0^L f(x)dx$ if f is even on $[-L, L]$.

13.3

Use a convergence theorem to determine the sum of the Fourier series of the function on the interval.

$$6. f(x) = \begin{cases} \cos(x) & \text{for } -2 \leq x < 0 \\ \sin(x) & \text{for } 0 \leq x \leq 2 \end{cases}$$

$$7. f(x) = \begin{cases} -1 & \text{for } -4 \leq x < 0 \\ 1 & \text{for } 0 \leq x \leq 4 \end{cases}$$

16. Let $f(x) = \frac{x^2}{2}$ for $-\pi \leq x \leq \pi$. Find the Fourier series of $f(x)$ and evaluate it

at an appropriately chosen value of x to sum the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

17. Use the Fourier series of Problem 16 to sum the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

13.4

Write the Fourier cosine series and the Fourier sine series of the function on the interval. Determine the sum of each series.

$$2. f(x) = \begin{cases} 1 & \text{for } 0 \leq x < \pi \\ -1 & \text{for } \pi < x \leq 2\pi \end{cases}$$

13. Let $f(x)$ be defined on $[-L, L]$. Prove that f can be written as the sum of an even and an odd function on this interval.

15. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(4n^2 - 1)}$. Hint: Expand $\sin(x)$ in a cosine series on $[0, \pi]$ and choose an appropriate value of x .