國立台灣海洋大學 2005 河工系工程數學(二) 第一次作業解答

13.2

5. $f(x)=\frac{16}{\pi}\sum_{n=1}^{\infty}(\frac{1}{2n-1})\sin[(2n-1)x]$

9. $f(x)=\frac{3}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}(\frac{1}{2n-1})\sin[(2n-1)x]$

14.Let f be even on $[-L,L]$, Then $\int_{-L}^{L}f(x)dx=\int_{-L}^{0}f(x)dx+\int_{0}^{L}f(x)dx$. In the first integral on the right, let $t=-x$ to get

$\int_{-L}^{L}f(x)dx=-\int_{L}^{0}f(-t)dt+\int_{0}^{L}f(x)dx=\int_{0}^{L}f(-t)dt+\int_{0}^{L}f(x)dx$.

But $f(-t)=f(t)$ since f is even. Hence $\int_{-L}^{L}f(x)dx=2\int_{0}^{L}f(x)dx$.

15. Let f be odd on $[-L,L]$, Then $\int_{-L}^{L}f(x)dx=\int_{-L}^{0}f(x)dx+\int_{0}^{L}f(x)dx$. let $t=-x$ to get $\int_{-L}^{L}f(x)dx=-\int_{L}^{0}f(-t)dt+\int_{0}^{L}f(x)dx=-\int_{0}^{L}f(t)dt+\int_{0}^{L}f(x)dx=0$

13.3

6. $f(x)=\frac{1}{4}[1+\sin(2)-\cos(2)]+\sum_{n=1}^{\infty}\frac{(-1)^n 2\sin(2)}{\pi^2 n^2-4}\cos(\frac{n\pi}{2}x)+$

$\sum\frac{(-1)^{n+1}n\pi}{n^2\pi^2-4}[1+(-1)^n(\sin(2)-\cos(2))]\sin(\frac{n\pi}{2}x)$

7. $f(x)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\sin[\frac{(2n-1)\pi}{4}x]$

16. $f(x)=\frac{x^2}{2}=\frac{\pi^2}{6}+2\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nx)$; Let $x=\pi$ get $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.

17. $f(x)=\frac{x^2}{2}=\frac{\pi^2}{6}+2\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nx)$; Let $x=0$ get $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}=\frac{-\pi^2}{12}$.

13.4

2. $c(x)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)}\cos[\frac{(2n-1)\pi}{2}x]$, $s(x)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)}\sin[\frac{(2n-1)\pi}{2}x]$.

13.Define $e(x)=\frac{1}{2}[f(x)+f(-x)]$ and $o(x)=\frac{1}{2}[f(x)-f(-x)]$. Then $e(x)+o(x)=f(x)$, and $e(x)=e(-x)$, and $o(x)=-o(-x)$.

15. $\sin(x)=\frac{2}{\pi}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos(2nx)$, Let $x=\frac{\pi}{2}$ to get $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}=\frac{1}{2}-\frac{\pi}{4}$