國立台灣海洋大學九十三學年度 第二學期 工程數學(二) 第二次期中考

1. $f(t)=H(t)e^{-at}$, where H(t) is the Heavside function, a>0.

(1) Compute $F(\omega)$. 5%

(2) Compute the **Fourier transform** of $e^{-a|t|}$ by using the result of (1). 7%

(Hint: a. $e^{-a|t|}=H(t)e^{-at}+H(-t)e^{at}$, b. Time reversal)

(3) $g(t)=H(t-4)e^{-2t}\sin(t-4)$, find $G(\omega)$ by using the result of (1) and the theorems of modulation and time shifting. 7%

(4)Proof $\mathscr{F}[\delta(t)]=1$. 7%

(5) Solving the y_p of $y''(t)+3y'(t)+2y(t)=\delta(t-5)$. 7%

(6) Solving the y_p of $y''(t)+3y'(t)+2y(t)=H(t)e^{-4t}$. 8%

(7) Solving the y_p of $y''(t)+3y'(t)+2y(t)=H(t)e^{-4t}*\delta(t)$, where $*$ is convolution operator. 4%

2. Proof the relations of $F(\omega)$ and $A(\omega), B(\omega)$, where $F(\omega)$ is the **Fourier transform of** $f(t)$, and $A(\omega), B(\omega)$ are the **Fourier integral coefficients** of $f(t)$. 15%

3. Let $f(t)=\begin{cases} k & for \quad -c\le t<c \\ 0 & for \quad t<-c \quad and \quad t\ge c \end{cases}$ and c, k are positive numbers

a) sketch the function $f(t)$ 3 %

b) is the function $f(t)$ piecewise smooth 2%

c) show that $\int_{-\infty}^{\infty}|f(t)|dt$ converges 5%

d) Find the Fourier integral coefficients, A_ω, B_ω, of $f(t)$ 12%

e) Find the Fourier integral of $f(t)$, $\int_0^\infty [A_\omega\cos(\omega t)+B_\omega\sin(\omega t)]\,d\omega$ (3 %) and determine what this integral converges to 5%

f) if $k=\dfrac{1}{2c}$ and $c\to 0$, find the Fourier integral of $f(t)$ 5%

4. Let $f(x)=xe^{-|x|}$, $g(x)=|x|e^{-|x|}$

a) Find the Fourier transform of $f(x)$ 12%

b) Find the Fourier transform of $g(x)$ 13%