# 海洋大學河海工程學系九十四學年度 第二學期 ■期中 □小考 考試命題紙

| 考 試 科 目 | 開課系級 | 考試日期 | 印製份數 | 答案紙 | 命題教師 | 備 註 |
|---|---|---|---|---|---|---|
| 工程數學二 | 二 A, B | 4 月 14 日 | 111 | ■ 需 □不需 | 陳桂鴻 呂學育 | **第一次大考** |

**1.** $\vec{F} = \dfrac{-y}{x^2+y^2}\vec{i} + \dfrac{x}{x^2+y^2}\vec{j}$; Evaluate $\oint_C \vec{F}d\vec{r}$

(a)C is shown as Fig1(a). (Hint: Using direct integral) (7%)

(b)C is shown as Fig1(b). (Hint: Using Green's theorem) (7%)

(d) C is shown as Fig1(c). (Hint: Using Green's theorem) (6%)

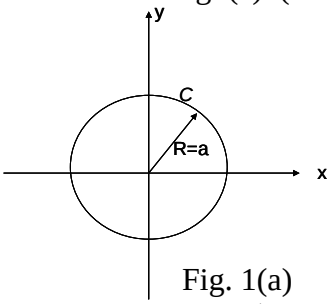


Fig. 1(a)

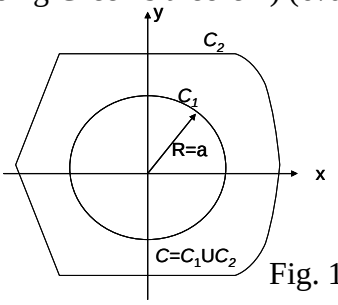


Fig. 1(b)

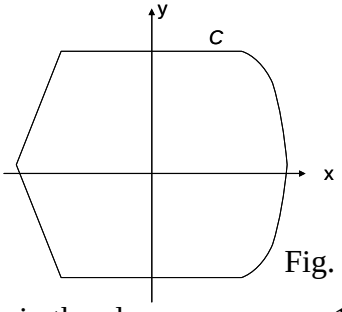


Fig. 1(c)

**2.** $\vec{F} = y^3\vec{i} + -x^3\vec{j} + z^3\vec{k}$;C is the trace of the cylinder $x^2+y^2=1$ in the plane $x+y+z=1$.

(a) Show that the force is conservative or nonconservative. (5%)

(b) Use Stokes's theorem to evaluate $\oint_C \vec{F}d\vec{r}$ . (15%)

**3.** The given vector field $\vec{F}(x,y,z) = (x\vec{i} + y\vec{j} + z\vec{k})/(x^2+y^2+z^2)$, *S* is the region bounded by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.

(a). Find $\nabla\cdot\vec{F}, \nabla\times\vec{F}$ . (5%)

(b). Find the normal vector $\vec{n}$ of *S*. (5%)

(c). Use the divergence theorem to find the outward flux $\iint_S (\vec{F}\cdot\vec{n})dS$ of $\vec{F}$ . (10%)

**4.** Suppose $\vec{r}(t) = t^2\vec{i} + (t^3-2t)\vec{j} + (t^2-5t)\vec{k}$ is the position vector of a moving particle. What are its speed, velocity, acceleration, curvature and tangent line at the point (0,0,0) ? (15 scores)

**5.** If $S$ is the portion of the plane $x+2y+3z=6$ in the first octant. For $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$

(1) find the area of $S$ (5 scores)

(2) find the upper unit normal of $S$ (5 scores)

(3) use Stokes' theorem to evaluate $\oint_C \vec{F}\cdot d\vec{r}$ ,where the curve $C$ is the boundary of $S$ and $C$ is oriented counterclockwise as viewed from above. (10 scores)

**6.** If $S$ is the surface of the region bounded by $x^2+y^2=4$, $z=\sqrt{16-x^2-y^2}$, $z=0$.

$\vec{F} = -y^3\vec{i} - x^3\vec{j} + z^3\vec{k}$

(1) find the volume of the solid bounded by $x^2+y^2=4$, $z=\sqrt{16-x^2-y^2}$, $z=0$. (10 scores)

(2) use the divergence theorem to find the outward flux $\iint_S (\vec{F}\cdot\vec{n})\,dS$ (15 scores)