

HOMEWORK #1 (Chapter 9 Vector Calculus)

(1). Suppose $\nabla f(a,b) = 4\mathbf{i} + 3\mathbf{j}$. Find a unit vector \mathbf{u} so that: (Exercises 9.5 problem 33).

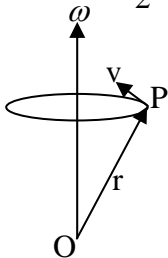
- (a). $D_{\mathbf{u}}f(a,b) = 0$.
- (b). $D_{\mathbf{u}}f(a,b)$ is a maximum.
- (c). $D_{\mathbf{u}}f(a,b)$ is a minimum.

(2). Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ be the position vector of a mass m_1 and let the mass m_2 be

located at the origin. If the force of gravitational attraction is $\mathbf{F} = -\frac{Gm_1m_2}{\|\mathbf{r}\|^3}\mathbf{r}$, verify that $\text{curl } \mathbf{F} = \mathbf{0}$ and $\text{div } \mathbf{F} = 0$, $\mathbf{r} \neq \mathbf{0}$. (Exercises 9.7 problem 39).

(3). Suppose a body rotates with a constant angular velocity $\boldsymbol{\omega}$ about an axis. If \mathbf{r} is the position vector of a point P on the body measured from the origin, then the linear velocity vector \mathbf{v} of rotation is $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$. See Figure. If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\boldsymbol{\omega} = \omega_1\mathbf{i} + \omega_2\mathbf{j} + \omega_3\mathbf{k}$,

show that $\boldsymbol{\omega} = \frac{1}{2} \text{curl } \mathbf{v}$. (Exercises 9.7 problem 40).



In problem 4, Find the length of the curve traced by the given vector function on the indicated interval. (Exercises 9.1 problem 44).

(4). $\mathbf{r}(t) = 3t\mathbf{i} + \sqrt{3}t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k}$; $0 \leq t \leq 1$.

In problem 5-6, evaluate $\oint_C (x^2 + y^2)dx - 2xydy$ on the given closed curve C. (Exercises 9.8 problem 19 and 20).

