HOMEWORK #1 (Chapter 9 Vector Calculus)

(1). Suppose $\nabla f(a,b)=4\underset{\sim}{i}+3\underset{\sim}{j}$. Find a unit vector **u** so that: (Exercises 9.5 problem 33).

(a). $D_u f(a,b)=0$.
(b). $D_u f(a,b)$ is a maximum.
(c). $D_u f(a,b)$ is a minimum.

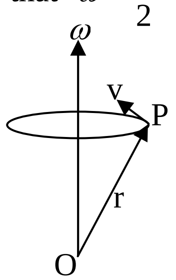
Ans: (a) **u**= $\pm(3\underset{\sim}{i}-4\underset{\sim}{j})/\sqrt{(9+16)}$ = $\pm(\frac{3}{5}\underset{\sim}{i}-\frac{4}{5}\underset{\sim}{j})$, (b) **u**= $(4\underset{\sim}{i}+3\underset{\sim}{j})/\sqrt{(9+16)}$ = $(\frac{4}{5}\underset{\sim}{i}+\frac{3}{5}\underset{\sim}{j})$,

(c) **u**= $-(\frac{4}{5}\underset{\sim}{i}+\frac{3}{5}\underset{\sim}{j})$

(2). Let $r=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$ be the position vector of a mass m_1 and let the mass m_2 be located at the origin. If the force of gravitational attraction is $F=-\frac{Gm_1m_2}{\|r\|^3}r$, verify that curl $F=0$ and div $F=0$, $r\neq 0$. (Exercises 9.7 problem 39).

Ans: $\text{curl}\,F=-Gm_1m_2\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{|r|^3} & \frac{y}{|r|^3} & \frac{z}{|r|^3} \end{vmatrix}=0$, $\text{div}\,F=0$

(3).Suppose a body rotates with a constant angular velocity ω about an axis. If **r** is the position vector of a point ***P*** on the body measured form the origin, then the linear velocity vector **v** of rotation is $v=\omega\times r$. See Figure. If $r=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$ and $\omega=\omega_1\underset{\sim}{i}+\omega_2\underset{\sim}{j}+\omega_3\underset{\sim}{k}$, show that $\omega=\frac{1}{2}$ curl **v**. (Exercises 9.7 problem 40).



Ans: $\frac{1}{2}$curl **v**= $\frac{1}{2}\text{curl}(\omega\times r)=\frac{1}{2}\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \omega_2 z-\omega_3 y & \omega_3 x-\omega_1 z & \omega_1 y-\omega_2 x \end{vmatrix}=\omega$

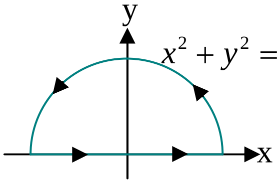
In problem 4, Find the length of the curve traced by the given vector function on the indicated interval. (Exercises 9.1 problem 44).

(4). $r(t) = 3t\,\underset{\sim}{i} + \sqrt{3}t^2\,\underset{\sim}{j} + \frac{2}{3}t^3\,\underset{\sim}{k}$; $0 \le t \le 1$.

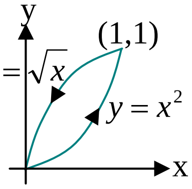
Ans: $r'(t) = 3\,\underset{\sim}{i} + 2\sqrt{3}t\,\underset{\sim}{j} + 2t^2\,\underset{\sim}{k}$, $\|r'(t)\| = 3 + 2t^2$, $s = \int_0^1 (3 + 2t^2)dt = \frac{11}{3}$

In problem 5-6, evaluate $\oint_C (x^2 + y^2)dx - 2xydy$ on the given closed curve C. (Exercises 9.8 problem 19 and 20).

(5).



(6).



Ans: (5) From (-2,0) to (2,0), $y = dy = 0$. Form (2,0) to (-2,0), $x = 2\cos\theta$, $y = 2\sin\theta$.

$$\oint_C (x^2 + y^2)dx - 2xydy = \int_{-2}^{2} x^2 dx + \int_0^{\pi} 4(-2\sin\theta)d\theta - \int_0^{\pi} 8\cos\theta\sin\theta(2\cos\theta)d\theta = \frac{-64}{3}$$

(6) $\oint_C (x^2 + y^2)dx - 2xydy$

$$= \int_0^1 (x^2 + x^4)dx - 2\int_0^1 xx^2(2xdx) + \int_1^0 (x^2 + x)dx - 2\int_1^0 x\sqrt{x}(\frac{x^{-1/2}}{2})dx = \frac{-3}{5}$$