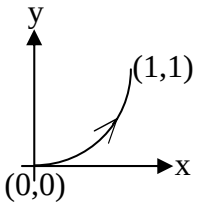
HOMEWORK #2 (Chapter 9 Vector Calculus)

(1). In this problem, find the work done by the force $F(x,y)=(2x+e^{-y})\underset{\sim}{i}+(4y-xe^{-y})\underset{\sim}{j}$ along the indicated curve. (Exercises 9.9 problem 17).



Ans: F is conservative and curve C: $y=x$, $0\le x\le 1$.

$$w=\int_C (2x+e^{-y})dx+(4y-xe^{-y})dy=\int_0^1 (2x+e^{-x})dx+\int_0^1 (4x-xe^{-x})dx$$

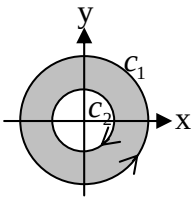
$$=3+e^{-1}$$

(2). The inverse square law of gravitational attraction between two masses m_1 and m_2 is given by $F=\dfrac{-Gm_1m_2r}{\|r\|^3}$, where $r=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$. Show the F is conservative. Find a potential function for F. (Exercises 9.9 problem 27).

Ans: $\because \nabla\times F=0$ $\therefore$ F is conservative. Potential function $\phi=\dfrac{Gm_1m_2}{|r|}$.

In problem (3) and (4), evaluate the given line integral where $c=c_1\cup c_2$ is the boundary of the shaded region R.

(3). $\oint_c (4x^2-y^3)dx+(x^3+y^2)dy$. $c_1: x^2+y^2=4$, $c_2: x^2+y^2=1$ (Exercises 9.12 problem 23).

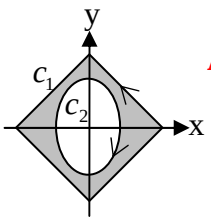


Ans: $P=4x^2-y^3$, $P_y=-3y^2$; $Q=x^3+y^2$, $Q_x=3x^2$

$$\oint_c (4x^2-y^3)dx+(x^3+y^2)dy=\iint_R (3x^2+3y^2)dA$$

$$=\int_0^{2\pi}\int_1^2 3r^2(rdrd\theta)=\frac{45\pi}{2}$$

(4). $\oint_c (\cos x^2-y)dx+(\sqrt{y^3+1})dy$. $c_2: 4x^2+y^2=16$ (Exercises 9.12 problem 24)

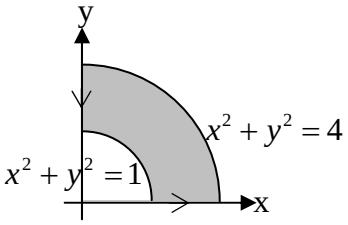


Ans: $P=\cos x^2-y$, $P_y=-1$; $Q=\sqrt{y^3+1}$, $Q_x=0$

$$\oint_c (\cos x^2-y)dx+(\sqrt{y^3+1})dy=\iint_R (0+1)dA=72-8\pi$$

(5). Find the work done by the force $F = -y\underset{\sim}{i} + x\underset{\sim}{j}$ acting along the cardioid $r = 1 + \cos\theta$.

(Exercises 9.12problem 33)



Ans: $W = \oint_C F \cdot dr = \oint_C -ydx + xdy = \iint_R 2dA$

$= 2\int_0^{2\pi}\int_0^{1+\cos\theta} rdrd\theta = 3\pi$