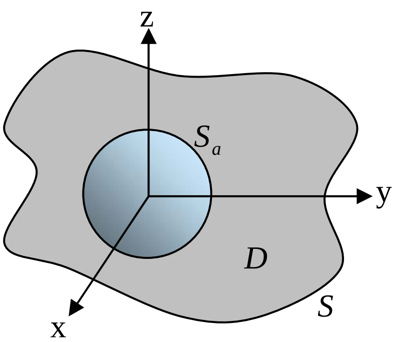
HOMEWORK #4 (Chapter 9 Vector Calculus)

(1). In this problem, use the divergence theorem to find the outward flux $\iint_S (F \cdot n)dS$ of the given vector field F. $F(x, y, z) = (x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k})/(x^2 + y^2 + z^2)$; D the region bounded by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ (Exercises 9.16 problem 9).

(2). The electric field at a point $P(x, y, z)$ due to a point charge q located at the origin is given by the inverse square field $E = q\dfrac{r}{\|r\|^3}$, where $r = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$. (Exercises 9.16 problem 15)

(a). Suppose S is a closed surface, S_a is a sphere $x^2 + y^2 + z^2 = a^2$ lying completely within S, and D is the region bounded between S and S_a. See Figure. Show that the outward flux of E for the region D is zero.



(b). Use the result of part (a) to prove Gauss' law: $\iint_S (E \cdot n)dS = 4\pi q$, that is the outward flux of the electric field E through any closed surface (for which the divergence theorem applies) containing the origin is $4\pi q$.