HOMEWORK #5 (Chapter 12 Orthogonal functions and Fourier series)

(1). $y^{(4)} + \lambda y'' = 0$, $y(0) = y''(0) = 0$, $y(2) = y'(2) = 0$. Find the general solution.

Ans: Let $\lambda = -k^2$ and $k \neq 0$

$y(x) = c_1 + c_2 x + c_3 \cosh kx + c_4 \sinh kx$

$y(0) = y''(0) = 0 \rightarrow c_1 = c_3 = 0$

$y(2) = y'(2) = 0 \rightarrow c_2 = c_4 = 0$

$\therefore \lambda = -k^2$不合

Let $\lambda = 0$

$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3$

$y(0) = y''(0) = 0 \rightarrow c_1 = c_3 = 0$

$y(2) = y'(2) = 0 \rightarrow c_2 = c_4 = 0$

$\therefore \lambda = 0$不合

Let $\lambda = k^2$

$y(x) = c_1 + c_2 x + c_3 \cos kx + c_4 \sin kx$

$y(0) = y''(0) = 0 \rightarrow c_1 = c_3 = 0$

$\therefore y(x) = c_2 x + c_4 \sin kx$

$y(2) = 0 \rightarrow 2c_2 + c_4 \sin 2k = 0$

$y'(2) = 0 \rightarrow c_2 + kc_4 \cos 2k = 0$

$\because c_2 \neq 0$ and $c_4 \neq 0$ $\therefore \begin{vmatrix} 2 & \sin 2k \\ 1 & k\cos 2k \end{vmatrix} = 0 \rightarrow \tan 2k = 2k$

設 $\tan 2k = 2k$ 的解爲 k_1, $k_2 \cdots k_n$

$\therefore y(x) = \sin k_n x - \dfrac{x}{2} \sin 2k_n$