HOMEWORK #6 (Chapter 12 Orthogonal functions and Fourier series)

In Problem 1-2, find the Fourier series of f on the given interval.

(1). $f(x)=\begin{cases}0, & -2<x<0\\ x, & 0\le x<1\\ 1, & 1\le x<2\end{cases}$ (Exercises 12.2 problem 12).

Ans: $a_0=\frac{1}{2}\int_{-2}^{2}f(x)dx=\frac{3}{4}$, $a_n=\frac{1}{2}\int_{-2}^{2}f(x)\cos\frac{n\pi}{2}xdx=\frac{2}{n^2\pi^2}(\cos\frac{n\pi}{2}-1)$

$b_n=\frac{1}{2}\int_{-2}^{2}f(x)\sin\frac{n\pi}{2}xdx=\frac{2}{n^2\pi^2}(\sin\frac{n\pi}{2}+\frac{n\pi}{2}(-1)^{n+1})$,

$$f(x)=\frac{3}{8}+\sum_{n=1}^{\infty}\left[\frac{2}{n^2\pi^2}(\cos\frac{n\pi}{2}-1)\cos\frac{n\pi}{2}x+\frac{2}{n^2\pi^2}(\sin\frac{n\pi}{2}+\frac{n\pi}{2}(-1)^{n+1})\sin\frac{n\pi}{2}x\right]$$

(2). $f(x)=e^x$, $-\pi<x<\pi$ (Exercises 12.2 problem 15).

Ans: $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}(e^{\pi}-e^{-\pi})$, $a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx=\frac{(-1)^n(e^{\pi}-e^{-\pi})}{(1+n^2)\pi}$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx=\frac{(-1)^n n(e^{-\pi}-e^{\pi})}{(1+n^2)\pi}$,

$$f(x)=\frac{(e^{\pi}-e^{-\pi})}{2\pi}+\sum_{n=1}^{\infty}\left[\frac{(-1)^n(e^{\pi}-e^{-\pi})}{(1+n^2)\pi}\cos nx+\frac{(-1)^n n(e^{-\pi}-e^{\pi})}{(1+n^2)\pi}\sin nx\right]$$

(3). Find the Fourier series of $f(x)=x,\ x\in[-\pi,\pi]$ and plot.

Ans: $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=0$, $a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx=0$, $b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx=\frac{-2(-1)^n}{n}$,

$$f(x)=\sum_{n=1}^{\infty}\left[\frac{-2(-1)^n}{n}\sin nx\right]$$