HOMEWORK #7 (Chapter 12 Orthogonal functions and Fourier series)

In Problem 1-2, find the half-range cosine and sine expansions of the given function.

(1). $f(x)=\begin{cases}0, & 0<x<1/2 \\ 1, & 1/2\le x<1\end{cases}$ (Exercises 12.3 problem 26).

Ans: $a_0=2\int_{1/2}^{1} f(x)dx=1$, $a_n=2\int_{1/2}^{1} f(x)\cos n\pi x dx=-\frac{2}{n\pi}\sin\frac{n\pi}{2}$,

$b_n=2\int_{1/2}^{1} f(x)\sin n\pi x dx=\frac{2}{n\pi}(\cos\frac{n\pi}{2}+(-1)^{n+1})$,

$f(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\left[-\frac{2}{n\pi}\sin\frac{n\pi}{2}\cos n\pi x\right]$; $f(x)=\sum_{n=1}^{\infty}\left[\frac{2}{n\pi}(\cos\frac{n\pi}{2}+(-1)^{n+1})\sin n\pi x\right]$

(2). $f(x)=\sin x,\ 0<x<\pi$ (Exercises 12.3 problem 28).

Ans: $a_0=\frac{2}{\pi}\int_0^{\pi} f(x)dx=\frac{4}{\pi}$, $a_n=\frac{2}{\pi}\int_0^{\pi} f(x)\cos n\pi x dx=\frac{2[(-1)^n+1]}{(1-n^2)\pi}$ for $n=2,3,4\cdots$,

$b_n=\frac{2}{\pi}\int_0^{\pi} f(x)\sin n\pi x dx=0$ for $n=2,3,4\cdots$,

$a_1=\frac{1}{\pi}\int_0^{\pi}\sin 2x dx=0$, $b_1=\frac{2}{\pi}\int_0^{\pi}\sin^2 x dx=1$,

$f(x)=\sin x$; $f(x)=\frac{2}{\pi}+\frac{2}{\pi}\sum_{n=2}^{\infty}\left[\frac{(-1)^n+1}{1-n^2}\cos nx\right]$