HOMEWORK #8 (Chapter 12 Orthogonal functions and Fourier series)

(1). Suppose a uniform beam of length L is simply supported at x=0 and at x=L. If the load per unit length is given by $w(x)=w_0 x/L$, $0<x<L$, then the differential equation for the deflection $y(x)$ is $EI\dfrac{d^4y}{dx^4}=\dfrac{w_0 x}{L}$, where E, I, and w_0 are constants.

(a) Expand $w(x)$ in a half-range sine series.

(b) Use the method of Example 4 to find a particular solution $y(x)$ of the differential equation. (Exercises 12.3 problem 45).

Ans: (a) $b_n=\dfrac{2}{L}\displaystyle\int_0^L \frac{w_0 x}{L}\sin\frac{n\pi}{L}x\,dx=\frac{2w_0}{n\pi}(-1)^{n+1}$

$$w(x)=\sum_{n=1}^{\infty}\frac{2w_0}{n\pi}(-1)^{n+1}\sin\frac{n\pi}{L}x$$

(b) Let $y=\displaystyle\sum_{n=1}^{\infty}B_n\sin\frac{n\pi}{L}x$

$$\because EIy^{(4)}=w(x),\qquad \therefore B_n=\frac{2w_0(-1)^{n+1}L^4}{EIn^5\pi^5};\qquad y=\sum_{n=1}^{\infty}\frac{2w_0(-1)^{n+1}L^4}{EIn^5\pi^5}\sin\frac{n\pi}{L}x$$

(2). Proceed as in Problem 1 to find a particular solution $y(x)$ (Exercises 12.3 problem 46).

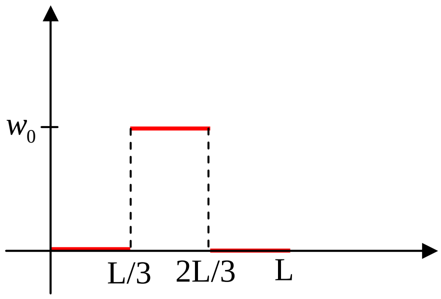

Figure 12.16

when the load per unit length is as given in Figure 12.16.

Ans: $b_n=\dfrac{2}{L}\displaystyle\int_{L/3}^{2L/3} w_0\sin\frac{n\pi}{L}x\,dx=\frac{2w_0}{n\pi}\left[\cos\frac{n\pi}{3}-\cos\frac{2n\pi}{3}\right]$

$$w(x)=\sum_{n=1}^{\infty}\frac{2w_0}{n\pi}\left[\cos\frac{n\pi}{3}-\cos\frac{2n\pi}{3}\right]\sin\frac{n\pi}{L}x$$

Let $y=\displaystyle\sum_{n=1}^{\infty}B_n\sin\frac{n\pi}{L}x$

$$\because EIy^{(4)}=w(x),\quad \therefore B_n=\frac{2w_0L^4}{EIn^5\pi^5}\left[\cos\frac{n\pi}{3}-\cos\frac{2n\pi}{3}\right];$$

$$y=\frac{2w_0L^4}{EI\pi^5}\sum_{n=1}^{\infty}\frac{1}{n^5}\left[\cos\frac{n\pi}{3}-\cos\frac{2n\pi}{3}\right]\sin\frac{n\pi}{L}x$$