# <u>Complex Numbers</u>

The complex numbers are the field $\mathbb{C}$ of numbers of the form $x+iy$, where *x* and *y* are real numbers and $i$ is the imaginary unit equal to the square root of -1, namely $i=\sqrt{-1}$.

We can apply a single letter $z=x+iy$ to denote a complex number. In component notation, $z=x+iy$ can be written (*x*, *y*).

Through the Euler formula, a complex number $z=x+iy$ may be written in "polar" form

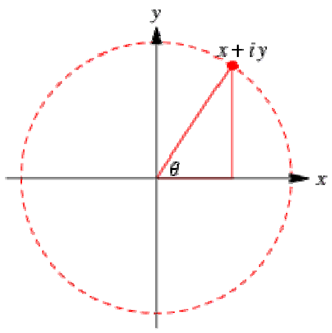
$$z=x+iy=r[\cos(\theta)+i\sin(\theta)]=re^{i\theta}$$

$r=|z|=|x+iy|=\sqrt{x^2+y^2}$ is the magnitude, or modulus, of $z=x+iy$

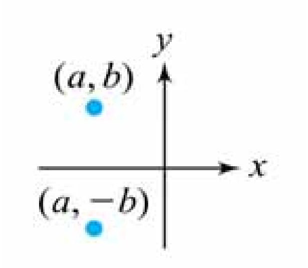
$\theta=\tan^{-1}(\frac{y}{x})$ is known as the argument or phase .

The plot below shows the point *z*, where the dashed circle represents the complex modulus

$r=|z|=|x+iy|=\sqrt{x^2+y^2}$ of *z* and the angle $\theta$ represents its complex argument.




Given a complex number $z=a+ib$, its conjugate of is $\bar{z}=a-ib$




**<u>Complex conjugate as a reflection across the horizontal axis</u>**

Complex addition

$$(a+bi)+(c+di)=(a+c)+i(b+d),$$

complex subtraction

$$(a+bi)-(c+di)=(a-c)+i(b-d),$$

complex multiplication

$$(a+bi)(c+di)=(ac-bd)+i(ad+bc),$$

and complex division

$$\frac{a+bi}{c+di}=\frac{(ac+bd)+i(bc-ad)}{c^2+d^2}$$

http://mathworld.wolfram.com/ComplexNumber.html

Equality of complex numbers

$a+ib=c+id$ if and only if $a=c$ $\quad and \quad$ $b=d$

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$e^{i\theta}=\cos\theta+i\sin\theta$ , $e^{-i\theta}=\cos\theta-i\sin\theta$

➔ $\sin\theta=\dfrac{e^{i\theta}-e^{-i\theta}}{2i}$ , $\cos\theta=\dfrac{e^{i\theta}+e^{-i\theta}}{2}$