Engineering Mathematics II Hw#1 日河工 2B

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Consider Example 13.8 in the textbook

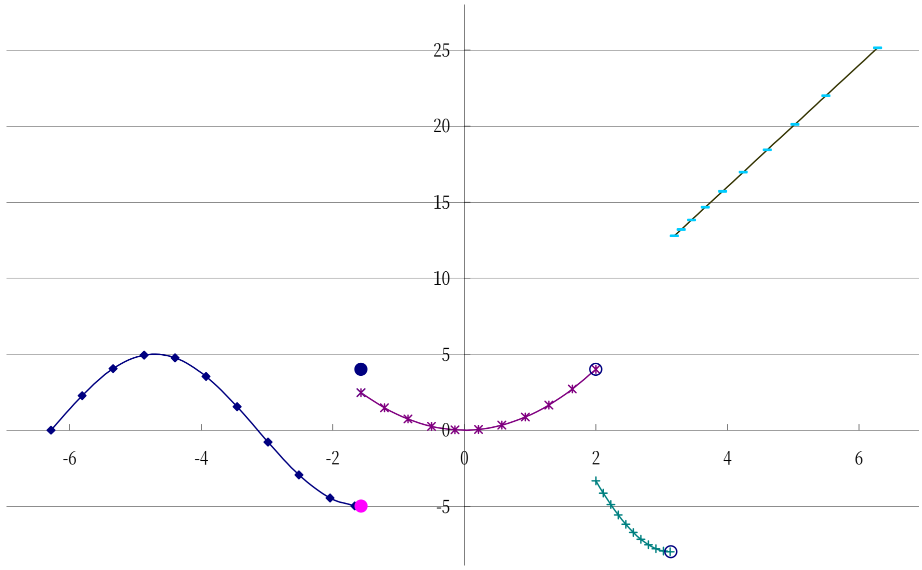
(a) Plot the function $f(x)$ on $[-2\pi,\ 2\pi]$

(b) Calculate the Fourier coefficients of $f(x)$ on $[-2\pi,\ 2\pi]$

(c) Plot $f(x)$ on $[-2\pi,\ 2\pi]$ and the Fourier series of $f(x)$ on $[-2\pi,\ 2\pi]$

(d) Discussions

(a)



Schematic graph of

$$f(x)=\begin{cases} 5\sin(x) & for\quad -2\pi\le x<-\pi/2 \\ 4 & for\quad x=-\pi/2 \\ x^2 & for\quad -\pi/2<x<2 \\ 8\cos(x) & for\quad 2\le x<\pi \\ 4x & for\quad \pi\le x\le 2\pi \end{cases}$$

(b)

$$a_n = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} f(x)\ \cos\left(\frac{nx}{2}\right)dx$$

$$= \frac{1}{2\pi}\left\{\int_{-2\pi}^{-\pi/2} 5\sin(x)\ \cos\left(\frac{nx}{2}\right)dx + \int_{-\pi/2}^{2} x^2\cos\left(\frac{nx}{2}\right)dx\right\} +$$

$$\frac{1}{2\pi}\left\{\int_{2}^{\pi} 8\cos(x)\ \cos\left(\frac{nx}{2}\right)dx + \int_{\pi}^{2\pi} 4x\cos\left(\frac{nx}{2}\right)dx\right\}$$

Hints:

$$\int \sin(px)\cos(qx)dx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)x}{2(p+q)}, \quad (p \neq \pm q)$$

$$\int \sin(ax)\cos(ax)dx = \frac{\sin^2(ax)}{2a}$$

$$\int x^2\cos(ax)dx = \frac{2x}{a^2}\cos(ax) + \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\sin(ax)$$

$$\int \cos(px)\cos(qx)dx = -\frac{\sin(p-q)x}{2(p-q)} + \frac{\sin(p+q)x}{2(p+q)}, \quad (p \neq \pm q)$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int x\cos(ax)dx = \frac{\cos(ax)}{a^2} + \frac{x\sin(ax)}{a}$$

(For $n \neq 2$)

$$= \frac{5}{2\pi}\left\{-\frac{\cos(1-\frac{n}{2})x}{2(1-\frac{n}{2})} - \frac{\cos(1+\frac{n}{2})x}{2(1+\frac{n}{2})}\right\}_{-2\pi}^{-\pi/2} + \frac{1}{2\pi}\left\{\frac{2x}{(\frac{n}{2})^2}\cos(\tfrac{nx}{2}) + \left(\frac{x^2}{\frac{n}{2}} - \frac{2}{(\frac{n}{2})^3}\right)\sin(\tfrac{n}{2}x)\right\}_{-\pi/2}^{2}$$

+

$$\frac{8}{2\pi}\left\{-\frac{\sin(1-\frac{n}{2})x}{2(1-\frac{n}{2})} + \frac{\sin(1+\frac{n}{2})x}{2(1+\frac{n}{2})}\right\}_{2}^{\pi} + \frac{4}{2\pi}\left\{\frac{\cos(\frac{nx}{2})}{(\frac{n}{2})^2} + \frac{x\sin(\frac{nx}{2})}{\frac{n}{2}}\right\}_{\pi}^{2\pi}$$

(For $n = 2$)

$$= \frac{5}{2\pi}\left\{\frac{\sin^2(\mathrm{x})}{2}\right\}_{-2\pi}^{-\pi/2} + \frac{1}{2\pi}\left\{\frac{2\mathrm{x}}{1}\cos(\mathrm{x}) + \left(\frac{\mathrm{x}^2}{1} - \frac{2}{1}\right)\sin(\mathrm{x})\right\}_{-\pi/2}^{2} +$$

$$\frac{8}{2\pi}\left\{\frac{\mathrm{x}}{2} + \frac{\sin(2\mathrm{x})}{4}\right\}_{2}^{\pi} + \frac{4}{2\pi}\left\{\frac{\cos(\mathrm{x})}{1} + \frac{\mathrm{x}\sin(\mathrm{x})}{1}\right\}_{\pi}^{2\pi}$$

$$b_n = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} f(x)\ \sin\left(\frac{nx}{2}\right)dx$$

$$= \frac{1}{2\pi}\left\{\int_{-2\pi}^{-\pi/2} 5\sin(x)\ \sin\left(\frac{nx}{2}\right)dx + \int_{-\pi/2}^{2} x^2 \sin\left(\frac{nx}{2}\right)dx\right\} +$$

$$\frac{1}{2\pi}\left\{\int_{2}^{\pi} 8\cos(x)\ \sin\left(\frac{nx}{2}\right)dx + \int_{\pi}^{2\pi} 4x\sin\left(\frac{nx}{2}\right)dx\right\}$$

Hints:

$$\int \sin(px)\sin(qx)dx = \frac{\sin(p-q)x}{2(p-q)} - \frac{\sin(p+q)x}{2(p+q)}, \quad (p \neq \pm q)$$

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int x^2\sin(ax)dx = \frac{2x}{a^2}\sin(ax) + \left(\frac{2}{a^3} - \frac{x^2}{a}\right)\cos(ax)$$

$$\int \sin(px)\cos(qx)dx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)x}{2(p+q)}, \quad (p \neq \pm q)$$

$$\int \sin(ax)\cos(ax)dx = \frac{\sin^2(ax)}{2a}$$

$$\int x\sin(ax)dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a}$$

(For $n \neq 2$)

$$= \frac{5}{2\pi}\left\{\frac{\sin(1-\frac{n}{2})x}{2(1-\frac{n}{2})} - \frac{\sin(1+\frac{n}{2})x}{2(1+\frac{n}{2})}\right\}_{-2\pi}^{-\pi/2} + \frac{1}{2\pi}\left\{\frac{2x}{(\frac{n}{2})^2}\sin(\tfrac{nx}{2}) + \left(\frac{2}{(\frac{n}{2})^3} - \frac{x^2}{\frac{n}{2}}\right)\cos(\tfrac{n}{2}x)\right\}_{-\pi/2}^{2} +$$

$$\frac{8}{2\pi}\left\{-\frac{\cos(\frac{n}{2}-1)x}{2(\frac{n}{2}-1)} - \frac{\cos(1+\frac{n}{2})x}{2(1+\frac{n}{2})}\right\}_{2}^{\pi} + \frac{4}{2\pi}\left\{\frac{\sin(\frac{nx}{2})}{(\frac{n}{2})^2} - \frac{x\cos(\frac{nx}{2})}{\frac{n}{2}}\right\}_{\pi}^{2\pi}$$

(For $n = 2$)

$$= \frac{5}{2\pi}\left\{\frac{x}{2} - \frac{\sin(2x)}{4}\right\}_{-2\pi}^{-\pi/2} + \frac{1}{2\pi}\left\{\frac{2x}{1}\sin(x) + \left(\frac{2}{1} - \frac{x^2}{1}\right)\cos(x)\right\}_{-\pi/2}^{2} +$$

$$\frac{8}{2\pi}\left\{\frac{\sin^2(x)}{2}\right\}_{2}^{\pi} + \frac{4}{2\pi}\left\{\frac{\sin(x)}{1} - \frac{x\cos(x)}{1}\right\}_{\pi}^{2\pi}$$