

1) Let  $f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq \pi/2 \\ 2 & \text{for } \pi/2 < x \leq \pi \end{cases}$

- (a) Find the Fourier cosine series of  $f(x)$  on  $[0, \pi]$
- (b) Find the Fourier sine series of  $f(x)$  on  $[0, \pi]$
- (c) Plot the Fourier cosine series obtained in (a)
- (d) Plot the Fourier sine series obtained in (b)
- (e) Make comparisons between (a), (c) and (b), (d) (hint: convergence in the interval and at the endpoints, convergence rate, Gibbs Phenomenon,.....)

2) Let  $f(x) = \begin{cases} 0 & \text{for } -\pi \leq x \leq 0 \\ x & \text{for } 0 < x \leq \pi \end{cases}$  (Section 13.5 Problem 3.)

- (a) Write the Fourier series of  $f(x)$  on  $[-\pi, \pi]$  and show that this series converges to  $f(x)$  on  $(-\pi, \pi)$ .
- (b) Show that this series can be integrated term-by-term.
- (c) Use the results of (a) and (b) to obtain a trigonometric series expansion for

$$\int_{-\pi}^{\pi} f(x) dt \text{ on } [-\pi, \pi].$$