Engineering Mathematics Ⅱ Hw#2 日河工 2B

9 March 2005

1) Let $f(x)=\begin{cases}1 & for \quad 0\le x\le \pi/2\\ 2 & for \quad \pi/2<x\le\pi\end{cases}$ (Please refer to example 13.20)

(a)Find the Fourier cosine series of $f(x)$ on $[0, \pi]$

(b)Find the Fourier sine series of $f(x)$ on $[0, \pi]$

(c)Plot the Fourier cosine series obtained in (a)

(d)Plot the Fourier sine series obtained in (b)

(e)Make comparisons between (a), (c) and (b), (d) (hint: convergence in the interval and at the endpoints, convergence rate, Gibbs Phenomenon,…...)

2) Let $f(x)=\begin{cases}0 & for \quad -\pi\le x\le 0\\ x & for \quad 0<x\le\pi\end{cases}$ (Section 13.5 Problem 3.)

(a)Write the Fourier series of $f(x)$ on $[-\pi, \pi]$ and show that this series converges to $f(x)$ on $(-\pi, \pi)$.

(b)Show that this series can be integrated term-by-term.

(c)Use the results of (a) and (b) to obtain a trigonometric series expansion for $\int_{-\pi}^{x} f(x)\ dt$ on $[-\pi, \pi]$.

ANS

(a) $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ dx=\frac{1}{\pi}\int_{0}^{\pi} x\ dx=\frac{\pi}{2}$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\ dx=\frac{1}{\pi}\int_{0}^{\pi} x\cos(nx)\ dx$$

$$=\frac{1}{\pi}\left\{x\frac{\sin(nx)}{n}+\frac{\cos(nx)}{n^2}\right\}\Bigg|_0^{\pi}$$

$$=\frac{1}{\pi}\left\{-\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right\}$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\ dx=\frac{1}{\pi}\int_{0}^{\pi} x\sin(nx)\ dx$$

$$=\frac{1}{\pi}\left\{-x\frac{\cos(nx)}{n}+\frac{\sin(nx)}{n^2}\right\}\Bigg|_0^{\pi}$$

$$=-\frac{1}{n}(-1)^n$$

➔The Fourier series of f on $[-\pi, \pi]$ is

$$\frac{\pi}{4}+\sum_{n=1}^{\pi}\left[\frac{(-1)^n-1}{\pi n^2}\cos(nx)+\frac{(-1)^{n+1}}{n}\sin(nx)\right]$$

Since f is piecewise smooth (?), the Theorem 13.2 gives us that this series converges to $f(x)$ on $(-\pi,\quad \pi)$.

(b)Since f is continuous, hence piecewise continuous, the Theorem 13.5 gives us that this series can be integrated term-by-term.

(c)Use the results of (a) and (b) to obtain a trigonometric series expansion for

$\int_{-\pi}^{x} f(x)\ dt$ on $[-\pi,\quad \pi]$.

Now integrate the Fourier series term-by-term over $[-\pi,\quad x]$ to get

$\frac{\pi}{4}(x+\pi)+\sum_{n=1}^{\pi}\left[\frac{(-1)^n-1}{\pi n^3}\sin(nx)+\frac{(-1)^n}{n^2}\cos(nx)-\frac{1}{n^2}\right]$. But this series is not exactly the Fourier series expansion due to the terms $\frac{\pi}{4}x$ and $-\sum_{n=1}^{\infty}\frac{1}{n^2}$. But we can obtain that $-\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ and $x=\sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\sin(nx)$ on $[-\pi,\quad \pi]$. Therefore, the corresponding Fourier series for $\int_{-\pi}^{x} f(x)\ dt$

on $[-\pi,\quad \pi]$ is $\frac{\pi^2}{12}+\sum_{n=1}^{\pi}\left\{\left[\frac{\pi(-1)^n}{2n^3}+\frac{1}{n^3\pi}\left((-1)^n-1\right)\right]\sin(nx)+\frac{(-1)^n}{n^2}\cos(nx)\right\}$