Engineering Mathematics II Hw#5 日河工 2B

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1) Section 14.3 Problems 1.

$$f(x) = xe^{-|x|}$$

$$C_\omega = \int_{-\infty}^{\infty} xe^{-|x|}e^{-i\omega x}dx = \int_{-\infty}^{0} xe^{x}e^{-i\omega x}dx + \int_{0}^{\infty} xe^{-x}e^{-i\omega x}dx$$

$$= x\frac{e^{(1-i\omega)x}}{1-i\omega}\Big|_{-\infty}^{0} - \int_{-\infty}^{0}\frac{e^{(1-i\omega)x}}{1-i\omega}dx + x\frac{e^{-(1+i\omega)x}}{-(1+i\omega)}\Big|_{0}^{\infty} - \int_{0}^{\infty}\frac{e^{-(1+i\omega)x}}{-(1+i\omega)}dx$$

$$= -\frac{e^{(1-i\omega)x}}{(1-i\omega)^2}\Big|_{-\infty}^{0} - \frac{e^{-(1+i\omega)x}}{(1+i\omega)^2}\Big|_{0}^{\infty}$$

$$= -\frac{1}{(1-i\omega)^2} + \frac{1}{(1+i\omega)^2} = \frac{-4i\omega}{(1+\omega^2)^2}$$

➔ The complex Fourier integral of $f(x) = xe^{-|x|}$

$$\frac{1}{2}\left(f(x+) + f(x-)\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} C_\omega e^{i\omega x}d\omega = -\frac{2i}{\pi}\int_{-\infty}^{\infty}\frac{\omega}{(1+\omega^2)^2}e^{i\omega x}d\omega$$

Further, the function $f(x) = xe^{-|x|}$ is continuous for all x as shown in Example 14.2. So the complex Fourier integral converges to $f(x)$ for all x.

2) Section 14.3 Problems 11.

$$f(x) = 5[H(x-3) - H(x-11)]$$

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} 5[H(t-3) - H(t-11)]e^{-i\omega x}dx$$

$$= \int_{3}^{\infty} 5e^{-i\omega x}dx - \int_{11}^{\infty} 5e^{-i\omega x}dx = \frac{5e^{-i\omega x}}{-i\omega}\Big|_{3}^{\infty} - \frac{5e^{-i\omega x}}{-i\omega}\Big|_{11}^{\infty}$$

$$= \frac{5e^{-3i\omega}}{i\omega} - \frac{5e^{-11i\omega}}{i\omega} = 5\frac{e^{-3i\omega} - e^{-11i\omega}}{i\omega}$$

$$= 5e^{-7i\omega}\frac{e^{4i\omega} - e^{-4i\omega}}{i\omega} = 5e^{-7i\omega}\frac{2i\sin(4\omega)}{i\omega} = 10e^{-7i\omega}\frac{\sin(4\omega)}{\omega}$$

3) Section 14.3 Problems 19.

$f(t)=3e^{-4|t|}\cos(2t)$

By Theorem 14.6

$\hat{f}(\omega)=F[3e^{-4|t|}\cos(2t)](\omega)=\frac{1}{2}\left(F[3e^{-4|t|}](\omega+2)+F[3e^{-4|t|}](\omega-2)\right)$

By Example 14.4, $F[3e^{-4|t|}](\omega)=3\frac{8}{16+\omega^2}$

$$\Rightarrow \begin{aligned}\hat{f}(\omega)&=\frac{1}{2}\left(F[3e^{-4|t|}](\omega+2)+F[3e^{-4|t|}](\omega-2)\right)\\&=\frac{1}{2}\left(\frac{24}{16+(\omega+2)^2}+\frac{24}{16+(\omega-2)^2}\right)=\frac{12}{16+(\omega+2)^2}+\frac{12}{16+(\omega-2)^2}\end{aligned}$$

4) Section 14.3 Problems 21.

$f(t)=\sin(t)/(4+t^2)$

By Theorem 14.6

$\hat{f}(\omega)=F[\sin(t)/(4+t^2)](\omega)=\frac{1}{2}i\left(F[\frac{1}{4+t^2}](\omega+1)-F[\frac{1}{4+t^2}](\omega-1)\right)$

By Section 14.3 Problems 8, $F[\frac{1}{4+t^2}](\omega)=\frac{\pi}{2}e^{-2|\omega|}$

$$\Rightarrow \begin{aligned}\hat{f}(\omega)&=\frac{1}{2}i\left(F[\frac{1}{4+t^2}](\omega+1)-F[\frac{1}{4+t^2}](\omega-1)\right)\\&=\frac{1}{2}i\left(\frac{\pi}{2}e^{-2|\omega+1|}-\frac{\pi}{2}e^{-2|\omega-1|}\right)=\frac{i\pi}{4}\left[e^{-2|\omega+1|}-e^{-2|\omega-1|}\right]\end{aligned}$$

5) Section 14.3 Problems 23.

$f(t)=H(t-1)e^{-5(t+2)}$

By Example 14.6 $g(t)=H(t)e^{-at}$ $\Rightarrow$ $\hat{g}(\omega)=\frac{1}{a+i\omega}$

By Theorem 14.1 $F[g(t-t_0)](\omega)=e^{-i\omega t_0}\hat{g}(\omega)=e^{-i\omega t_0}\frac{1}{a+i\omega}$

$\Rightarrow f(t)=H(t-1)e^{-5(t+2)}=H(t-1)e^{-5(t-1)}e^{-15}$

$\Rightarrow \hat{f}(\omega)=F\left\{H(t-1)e^{-5(t-1)}e^{-15}\right\}=e^{-15}e^{-i\omega}\frac{1}{5+i\omega}=\frac{e^{-(15+i\omega)}}{5+i\omega}$

6) Section 14.3 Problems 27.

$$\hat{f}(\varpi) = \frac{e^{(2\varpi-6)i}}{5-(3-\varpi)i} = \frac{e^{-2(3-\varpi)i}}{5-(3-\varpi)i}$$

By Theorem 14.2 Frequency Shitting

$$F[e^{i\varpi_0 t} g(t)] = \hat{g}(\varpi-\varpi_0) \Rightarrow e^{i\varpi_0 t} g(t) = F^{-1}[\hat{g}(\varpi-\varpi_0)](t)$$

$$\Rightarrow \hat{f}(\varpi) = \frac{e^{(2\varpi-6)i}}{5-(3-\varpi)i} = \frac{e^{2(\varpi-3)i}}{5+(\varpi-3)i} = F\left[e^{3it}\frac{e^{2i\varpi}}{5+\varpi i}\right]$$

$$\Rightarrow f(t) = e^{3it} F^{-1}\left[\frac{e^{2i\varpi}}{5+\varpi i}\right]$$

By Theorem 14.1 Time Shitting and Example 14.12

$$F^{-1}\left[\frac{e^{2i\varpi}}{5+\varpi i}\right] = f(t+2) = H(t+2)e^{-5(t+2)}$$

$$\Rightarrow f(t) = e^{3it} F^{-1}\left[\frac{e^{2i\varpi}}{5+\varpi i}\right] = e^{3it} H(t+2)e^{-5(t+2)}$$

$$\Rightarrow f(t) = e^{3it} H(t+2)e^{-5(t+2)} = H(t+2)e^{-10-5t+3it}$$

7) Section 14.3 Problems 29.

$$\hat{f}(\varpi) = \frac{1+i\omega}{6-\varpi^2+5i\varpi} = \frac{1+i\omega}{(3+i\varpi)(2+i\varpi)} = \frac{a}{3+i\varpi} + \frac{b}{2+i\varpi}$$

$$\Rightarrow \frac{1+i\omega}{(3+i\varpi)(2+i\varpi)} = \frac{a}{3+i\varpi} + \frac{b}{2+i\varpi} = \frac{2a+ai\varpi+3b+bi\varpi}{(3+i\varpi)(2+i\varpi)}$$

$$\Rightarrow \begin{matrix} 2a+3b=1 \\ i\varpi = (a+b)i\varpi \end{matrix} \Rightarrow \begin{matrix} 2a+3b=1 \\ a+b=1 \end{matrix} \Rightarrow \begin{matrix} a=2 \\ b=-1 \end{matrix}$$

$$\hat{f}(\varpi) = \frac{2}{3+i\varpi} - \frac{1}{2+i\varpi}$$

$\Rightarrow$**By Example 14.9** $g(t) = H(t)e^{-at}$ *and* $\hat{g}(\varpi) = \frac{1}{a+i\varpi}$

$$\Rightarrow f(t) = 2H(t)e^{-3t} - H(t)e^{-2t}$$