Engineering Mathematics II Hw#6 日河工 2B

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1) Section 6.1 Problems 3.

$$A=\begin{pmatrix} x & 1-x \\ 2 & e^x \end{pmatrix},\quad B=\begin{pmatrix} 1 & -6 \\ x & \cos(x) \end{pmatrix}$$

➔

$$A^2+2AB=\begin{pmatrix} x & 1-x \\ 2 & e^x \end{pmatrix}\cdot\begin{pmatrix} x & 1-x \\ 2 & e^x \end{pmatrix}+2\begin{pmatrix} x & 1-x \\ 2 & e^x \end{pmatrix}\begin{pmatrix} 1 & -6 \\ x & \cos(x) \end{pmatrix}$$

$$=\begin{pmatrix} x\cdot x+(1-x)\cdot 2 & x\cdot(1-x)+(1-x)\cdot e^x \\ 2\cdot x+e^x\cdot 2 & 2\cdot(1-x)+e^x\cdot e^x \end{pmatrix}+$$

$$2\begin{pmatrix} x\cdot 1+(1-x)\cdot x & x\cdot(-6)+(1-x)\cdot\cos(x) \\ 2\cdot 1+e^x\cdot x & 2\cdot(-6)+e^x\cdot\cos(x) \end{pmatrix}$$

$$=\begin{pmatrix} x^2+2-2x & x-x^2+e^x-xe^x \\ 2x+2e^x & 2-2x+e^{2x} \end{pmatrix}+\begin{pmatrix} 4x-2x^2 & -12x+2\cos(x)-2x\cos(x) \\ 4+2xe^x & -24+2e^x\cos(x) \end{pmatrix}$$

$$=\begin{pmatrix} 2+2x-x^2 & -11x-x^2+e^x-xe^x+2\cos(x)-2x\cos(x) \\ 4+2x+2e^x+2xe^x & -22-2x+e^{2x}+2e^x\cos(x) \end{pmatrix}$$

2) Section 6.1 Problems 13.

$$A=\begin{pmatrix} -4 & -2 & 0 \\ 0 & 5 & 3 \\ -3 & 1 & 1 \end{pmatrix}_{3\times 3},\quad B=\begin{pmatrix} 1 & -3 & 4 \end{pmatrix}_{1\times 3}$$

➔ $A_{3\times 3}B_{1\times 3}$ is not defined. $B_{1\times 3}A_{3\times 3}=\begin{pmatrix} 1 & -3 & 4 \end{pmatrix}\begin{pmatrix} -4 & -2 & 0 \\ 0 & 5 & 3 \\ -3 & 1 & 1 \end{pmatrix}=\begin{pmatrix} -16 & -13 & -5 \end{pmatrix}$

3) Section 7.5 Problems 1.

$$A=\begin{pmatrix} -4 & 2 & -8 \\ 1 & 1 & 0 \\ 1 & -3 & 0 \end{pmatrix},\quad \begin{vmatrix} -4 & 2 & -8 \\ 1 & 1 & 0 \\ 1 & -3 & 0 \end{vmatrix}=(-1)^{1+3}(-8)\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix}=32$$

4) Section 7.5 Problems 7.

$$A=\begin{pmatrix} -3 & 1 & 14 \\ 0 & 1 & 16 \\ 2 & -3 & 4 \end{pmatrix},$$

$$\begin{vmatrix} -3 & 1 & 14 \\ 0 & 1 & 16 \\ 2 & -3 & 4 \end{vmatrix} = (-1)^{1+1}(-3)\begin{vmatrix} 1 & 16 \\ -3 & 4 \end{vmatrix} + (-1)^{3+1}(2)\begin{vmatrix} 1 & 14 \\ 1 & 16 \end{vmatrix} = -156 + 4 = -152$$

5) Section 7.7 Problems 1.

$A = \begin{pmatrix} 2 & -1 \\ 1 & 6 \end{pmatrix}$, ➔ $\begin{vmatrix} 2 & -1 \\ 1 & 6 \end{vmatrix} = 13 \neq 0$ ➔nonsingular

➔ $A^{-1} = \dfrac{1}{13}\begin{pmatrix} 6 & 1 \\ -1 & 2 \end{pmatrix}$

6) Section 7.7 Problems 5.

$A = \begin{pmatrix} 6 & -1 & 3 \\ 0 & 1 & -4 \\ 2 & 2 & -3 \end{pmatrix}$ ➔ $\begin{vmatrix} 6 & -1 & 3 \\ 0 & 1 & -4 \\ 2 & 2 & -3 \end{vmatrix} = -18 + 8 - 6 + 48 = 32 \neq 0$ ➔nonsingular

➔ $A^{-1} = \dfrac{1}{32}\begin{pmatrix} 5 & 3 & 1 \\ -8 & -24 & 24 \\ -2 & -14 & 6 \end{pmatrix}$

7) Section 8.2 Problems 1.

$A = \begin{pmatrix} 0 & -1 \\ 4 & 3 \end{pmatrix}$

$P_A(\lambda) = \begin{vmatrix} \lambda & 1 \\ -4 & \lambda - 3 \end{vmatrix} = (\lambda)(\lambda - 3) + 4 = \lambda^2 - 3\lambda + 4$

(1)Eigenvalues of A ➔ $P_A(\lambda) = \lambda^2 - 3\lambda + 4 = 0 ==> \lambda_1 = \dfrac{3+\sqrt{7}i}{2}, \quad \lambda_2 = \dfrac{3-\sqrt{7}i}{2}$

(2)Find **an** eigenvector associated with $\lambda_1 = \dfrac{3+\sqrt{7}i}{2}$

➔ $(\dfrac{3+\sqrt{7}i}{2} \cdot I_2 - A)X = \begin{pmatrix} \dfrac{3+\sqrt{7}i}{2} & 1 \\ -4 & \dfrac{-3+\sqrt{7}i}{2} \end{pmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = O$

➔ $\begin{matrix} \dfrac{3+\sqrt{7}i}{2}x_1 + x_2 = 0 \\ -4x_1 + \dfrac{-3+\sqrt{7}i}{2}x_2 = 0 \end{matrix}$ ➔ $x_1 = (-3+\sqrt{7}i)\alpha, \quad x_2 = 8\alpha$

$\Rightarrow V_1 = \begin{pmatrix} -3+\sqrt{7}i \\ 8 \end{pmatrix}$

Find **an** eigenvector associated with $\lambda_2 = \dfrac{3-\sqrt{7}i}{2}$

$$\Rightarrow \left(\frac{3-\sqrt{7}i}{2} \cdot I_2 - A\right)X = \begin{pmatrix} \frac{3-\sqrt{7}i}{2} & 1 \\ -4 & \frac{-3-\sqrt{7}i}{2} \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = O$$

$$\Rightarrow \begin{matrix} \frac{3-\sqrt{7}i}{2}x_1 + x_2 = 0 \\ -4x_1 + \frac{-3-\sqrt{7}i}{2}x_2 = 0 \end{matrix} \quad \Rightarrow x_1 = (-3-\sqrt{7}i)\alpha,\ \ x_2 = 8\alpha \ \Rightarrow V_2 = \begin{pmatrix} -3-\sqrt{7}i \\ 8 \end{pmatrix}$$

By the Theorem 8.6, the eigenvectors corresponding to A, with 2 distinct eigenvalues, are linearly independent.==> A is diagonalizable.

(3) $Q = [V_1 \quad V_2] = \begin{bmatrix} -3+\sqrt{7}i & -3-\sqrt{7}i \\ 8 & 8 \end{bmatrix}$

$$\Rightarrow Q^{-1}AQ = \begin{bmatrix} \frac{3+\sqrt{7}i}{2} & 0 \\ 0 & \frac{3-\sqrt{7}i}{2} \end{bmatrix}$$

8) Section 8.2 Problems 5.

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\Rightarrow P_A(\lambda) = \begin{vmatrix} \lambda-5 & 0 & 0 \\ -1 & \lambda & -3 \\ 0 & 0 & \lambda+2 \end{vmatrix} = (\lambda-5)(\lambda)(\lambda+2)$$

(1)Eigenvalues of A $\Rightarrow P_A(\lambda) = 0 \Longrightarrow \lambda_1 = 0,\quad \lambda_2 = 5,\quad \lambda_3 = -2$

(2)Find **an** eigenvector associated with $\lambda_1 = 0$

$$\Rightarrow (0 \cdot I_3 - A)X = \begin{pmatrix} -5 & 0 & 0 \\ -1 & 0 & -3 \\ 0 & 0 & 2 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O \ \Rightarrow \begin{matrix} -5x_1 = 0 \\ -x_1 - 3x_3 = 0 \\ 2x_3 = 0 \end{matrix}$$

$$\Rightarrow X = \begin{Bmatrix} 0 \\ \alpha \\ 0 \end{Bmatrix}, \Rightarrow V_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Find **an** eigenvector associated with $\lambda_2 = 5$

$$\Rightarrow (5 \cdot I_3 - A)X = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 5 & -3 \\ 0 & 0 & 7 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O \Rightarrow \begin{matrix} -x_1 + 5x_2 - 3x_3 = 0 \\ 7x_3 = 0 \end{matrix}$$

$$\Rightarrow X = \begin{Bmatrix} 5\alpha \\ \alpha \\ 0 \end{Bmatrix}, \Rightarrow V_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$$

Find **an** eigenvector associated with $\lambda_3 = -2$

$$\Rightarrow (-2 \cdot I_3 - A)X = \begin{pmatrix} -7 & 0 & 0 \\ -1 & -2 & -3 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O \Rightarrow \begin{matrix} -7x_1 = 0 \\ -x_1 - 2x_2 - 3x_3 = 0 \end{matrix}$$

$$\Rightarrow X = \begin{Bmatrix} 0 \\ 3\alpha \\ -2\alpha \end{Bmatrix}, \Rightarrow V_3 = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$$

By the Theorem 8.6, the eigenvectors corresponding to A, with 3 distinct eigenvalues, are linearly independent.==> A is diagonalizable.

$$P = [V_1 \quad V_2 \quad V_3] = \begin{bmatrix} 0 & 5 & 0 \\ 1 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow P^{-1} = -\frac{1}{10} \begin{bmatrix} 2 & -10 & 15 \\ -2 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$\Rightarrow P^{-1}AP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

9) Section 8.2 Problems 7.

$$A = \begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\Rightarrow P_A(\lambda)=\begin{vmatrix}\lambda+2 & 0 & -1\\ -1 & \lambda-1 & 0\\ 0 & 0 & \lambda+2\end{vmatrix}=(\lambda+2)(\lambda-1)(\lambda+2)$$

(1)Eigenvalues of A $\Rightarrow P_A(\lambda)=0 \Longrightarrow \lambda_1=1,\quad \lambda_2=-2,\quad \lambda_3=-2$

(2)Find **an** eigenvector associated with $\lambda_1=1$

$$\Rightarrow (1\cdot I_3-A)X=\begin{pmatrix}3 & 0 & -1\\ -1 & 0 & 0\\ 0 & 0 & 3\end{pmatrix}\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}=O \Rightarrow \begin{matrix}3x_1-x_3=0\\ -x_1=0\\ 3x_3=0\end{matrix}$$

$$\Rightarrow X=\begin{Bmatrix}0\\ \alpha\\ 0\end{Bmatrix},\ \Rightarrow V_1=\begin{pmatrix}0\\ 1\\ 0\end{pmatrix}$$

Find **an** eigenvector associated with $\lambda=-2$

$$\Rightarrow (-2\cdot I_3-A)X=\begin{pmatrix}0 & 0 & -1\\ -1 & -3 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}=O \Rightarrow \begin{matrix}-x_3=0\\ -x_1-3x_2=0\end{matrix}$$

$$\Rightarrow X=\begin{Bmatrix}-3\alpha\\ \alpha\\ 0\end{Bmatrix},\ \Rightarrow V_2=\begin{pmatrix}-3\\ 1\\ 0\end{pmatrix}$$

We can only obtain two (linearly independent) eigenvectors. Therefore, A is not diagonalizable.