Engineering Mathematics Ⅱ Hw#8 日河工 2B

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1) Section 8.2 Problems 15.

$$A=\begin{pmatrix}-1 & 0\\ 1 & -5\end{pmatrix}$$

$$P_A(\lambda)=\begin{vmatrix}\lambda+1 & 0\\ -1 & \lambda+5\end{vmatrix}=(\lambda+1)(\lambda+5)-4$$

(1)Eigenvalues of A ➔ $P_A(\lambda)=0 \Longrightarrow \lambda_1=-1,\quad \lambda_2=-5$

(2)Find **an** eigenvector associated with $\lambda_1=-1$

$$\Rightarrow (-1\cdot I_2-A)X=\begin{pmatrix}0 & 0\\ -1 & 4\end{pmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}=O \Rightarrow -x_1+4x_2=0$$

$$\Rightarrow x_2=\alpha,\quad x_1=4\alpha$$

$$\Rightarrow V_1=\begin{pmatrix}4\\ 1\end{pmatrix}$$

Find **an** eigenvector associated with $\lambda_2=-5$

$$\Rightarrow (5\cdot I_2-A)X=\begin{pmatrix}-4 & 0\\ -1 & 0\end{pmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}=O \Rightarrow \begin{matrix}-4x_1=0\\ -x_1=0\end{matrix}$$

$$\Rightarrow x_1=0,\quad x_2=\beta$$

$$\Rightarrow V_2=\begin{pmatrix}0\\ 1\end{pmatrix}$$

(3) V_1, V_2 are linearly independent eigenvectors (why ?)

$$Q=[V_1\quad V_2]=\begin{bmatrix}4 & 0\\ 1 & 1\end{bmatrix} \Rightarrow Q^{-1}=\frac{1}{4}\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$\Rightarrow Q^{-1}AQ=\frac{1}{4}\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}\begin{bmatrix}-1 & 0\\ 1 & 5\end{bmatrix}\begin{bmatrix}4 & 0\\ 1 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -5\end{bmatrix}$$

$$\Rightarrow Q^{-1}AQ\cdot Q^{-1}AQ\cdots Q^{-1}AQ=Q^{-1}A^{18}Q=\begin{bmatrix}-1 & 0\\ 0 & -5\end{bmatrix}^{18}=\begin{bmatrix}(-1)^{18} & 0\\ 0 & (-5)^{18}\end{bmatrix}$$

$$\Rightarrow A^{18}=Q\begin{bmatrix}(-1)^{18} & 0\\ 0 & (-5)^{18}\end{bmatrix}Q^{-1}=\begin{bmatrix}1 & 0\\ \frac{1-5^{18}}{4} & 5^{18}\end{bmatrix}$$

2) Section 8.4 Problems 13.

Consider $-2x_1x_2+2x_3^2=-x_1x_2-x_2x_1+2x_3^2=X^tAX$

➔ $X^tAX=\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 2\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}$

➔The eigenvalues of A are $\lambda=-1,\quad 1,\quad 2$

➔By the Theorem 8.13, the standard form of this quadratic form X^tAX is

$-y_1^2+y_2^2+2y_3^2$

3) Section 8.4 Problems 17.

Consider $4x_1^2-4x_2^2+6x_1x_2=8$
$\Rightarrow 4x_1^2-4x_2^2+3x_1x_2+3x_2x_1=X^tAX=8$

➔ $X^tAX=\begin{pmatrix}x_1 & x_2\end{pmatrix}\begin{bmatrix}4 & 3\\ 3 & -4\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}$ ➔ The eigenvalues of A are $\lambda=5,\quad -5$

The standard form of this quadratic form X^tAX is

$5y_1^2-5y_2^2=8$

4) Section 8.4 Problems 21.

$A=\begin{pmatrix}6 & 1 & -7\\ 1 & 2 & 0\\ -7 & 0 & 1\end{pmatrix}$ ➔ $X^tAX=\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}\begin{pmatrix}6 & 1 & -7\\ 1 & 2 & 0\\ -7 & 0 & 1\end{pmatrix}\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}$

➔ $6x_1^2+2x_1x_2-14x_1x_3+2x_2^2+x_3^2$

5) Section 8.5 Problems 3.

$S=\begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 1-i\\ 0 & -1-i & 0\end{pmatrix}$ ➔ $\overline{S}=\begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 1+i\\ 0 & -1+i & 0\end{pmatrix}, S^t=\begin{pmatrix}0 & -1 & 0\\ 1 & 0 & -1-i\\ 0 & 1-i & 0\end{pmatrix}$

➔ $\overline{S}=-S^t$ ➔ The matrix is skew-hermitian.

➔ $P_S(\lambda)=\lambda(\lambda^2+3)$, The eigenvalues of S are $\lambda=0,\quad \sqrt{3}i,\quad -\sqrt{3}i$

Find **an** eigenvector associated with $\lambda_1 = 0$

$$\Rightarrow (0 \cdot I_3 - S)X = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1+i \\ 0 & 1+i & 0 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O \Rightarrow \begin{matrix} -x_2 = 0 \\ x_1 + (-1+i)x_3 = 0 \\ (1+i)x_2 = 0 \end{matrix}$$

$$\Rightarrow x_1 = 2\alpha, \;\; x_2 = 0, \;\; x_3 = \alpha(1+i)$$

$$\Rightarrow V_1 = \begin{pmatrix} 2 \\ 0 \\ 1+i \end{pmatrix}$$

Find **an** eigenvector associated with $\lambda_2 = \sqrt{3}i$

$$\Rightarrow (\sqrt{3}i \cdot I_3 - S)X = \begin{pmatrix} \sqrt{3}i & -1 & 0 \\ 1 & \sqrt{3}i & -1+i \\ 0 & 1+i & \sqrt{3}i \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O \Rightarrow \begin{matrix} \sqrt{3}ix_1 - x_2 = 0 \\ x_1 + \sqrt{3}ix_2 + (-1+i)x_3 = 0 \\ (1+i)x_2 + \sqrt{3}ix_3 = 0 \end{matrix}$$

$$\Rightarrow x_1 = \alpha, \;\; x_2 = \sqrt{3}i\alpha, \;\; x_3 = -\alpha(1+i)$$

$$\Rightarrow V_2 = \begin{pmatrix} 1 \\ \sqrt{3}i \\ -1-i \end{pmatrix}$$

Find **an** eigenvector associated with $\lambda_2 = -\sqrt{3}i$

$$\Rightarrow (-\sqrt{3}i \cdot I_3 - S)X = \begin{pmatrix} -\sqrt{3}i & -1 & 0 \\ 1 & -\sqrt{3}i & -1+i \\ 0 & 1+i & -\sqrt{3}i \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O$$

$$\begin{matrix} -\sqrt{3}ix_1 - x_2 = 0 \\ \Rightarrow x_1 - \sqrt{3}ix_2 + (-1+i)x_3 = 0 \\ (1+i)x_2 - \sqrt{3}ix_3 = 0 \end{matrix} \Rightarrow x_1 = \alpha, \;\; x_2 = -\sqrt{3}i\alpha, \;\; x_3 = -\alpha(1+i)$$

$$\Rightarrow V_3 = \begin{pmatrix} 1 \\ -\sqrt{3}i \\ -1-i \end{pmatrix}$$

$\Rightarrow P = (V_1 \quad V_2 \quad V_3) \quad \Rightarrow P^{-1}SP = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{3}i & 0 \\ 0 & 0 & -\sqrt{3}i \end{pmatrix}$

6) Section 8.5 Problems 9.

$$H = \begin{pmatrix} 8 & -1 & i \\ -1 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \Rightarrow \overline{H} = \begin{pmatrix} 8 & -1 & -i \\ -1 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, H^t = \begin{pmatrix} 8 & -1 & -i \\ -1 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$\Rightarrow \overline{H} = H^t \Rightarrow$ The matrix is hermitian.

$\Rightarrow P_H(\lambda) = \lambda(\lambda^2 - 8\lambda - 2)$

The eigenvalues of H are $\lambda = 0, \quad 4+3\sqrt{2}, \quad 4-3\sqrt{2}$

Find **an** eigenvector associated with $\lambda_1 = 0$

$$\Rightarrow (0 \cdot I_3 - H)X = \begin{pmatrix} -8 & 1 & -i \\ 1 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O \Rightarrow \begin{matrix} -8x_1 + x_2 - ix_3 = 0 \\ x_1 = 0 \\ ix_1 = 0 \end{matrix}$$

$\Rightarrow x_1 = 0, \quad x_2 = \alpha i, \quad x_3 = \alpha$

$$\Rightarrow V_1 = \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix}$$

Find **an** eigenvector associated with $\lambda_2 = 4+3\sqrt{2}$

$$\Rightarrow ((4+3\sqrt{2}) \cdot I_3 - H)X = \begin{pmatrix} -4+3\sqrt{2} & 1 & -i \\ 1 & 4+3\sqrt{2} & 0 \\ i & 0 & 4+3\sqrt{2} \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O$$

$(-4+3\sqrt{2})x_1 + x_2 - ix_3 = 0$

$\Rightarrow x_1 + (4+3\sqrt{2})x_2 = 0$

$ix_1 + (4+3\sqrt{2})x_3 = 0$

$\Rightarrow x_1 = -\alpha(4+3\sqrt{2}), \quad x_2 = \alpha, \quad x_3 = \alpha i$

➔ $V_2 = \begin{pmatrix} 4+3\sqrt{2} \\ -1 \\ -i \end{pmatrix}$

Find **an** eigenvector associated with $\lambda_3 = 4-3\sqrt{2}$

➔ $((4-3\sqrt{2})\cdot I_3 - H)X = \begin{pmatrix} -4-3\sqrt{2} & 1 & -i \\ 1 & 4-3\sqrt{2} & 0 \\ i & 0 & 4-3\sqrt{2} \end{pmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = O$

$(-4-3\sqrt{2})x_1 + x_2 - ix_3 = 0$

➔ $x_1 + (4-3\sqrt{2})x_2 = 0$

$ix_1 + (4-3\sqrt{2})x_3 = 0$

➔ $x_1 = -\alpha(4-3\sqrt{2}),\ \ x_2 = \alpha,\ \ x_3 = \alpha i$

➔ $V_3 = \begin{pmatrix} 4-3\sqrt{2} \\ -1 \\ -i \end{pmatrix}$

➔ $P = \begin{pmatrix} V_1 & V_2 & V_3 \end{pmatrix}$ ➔ $P^{-1}HP = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4+3\sqrt{2} & 0 \\ 0 & 0 & 4-3\sqrt{2} \end{pmatrix}$