考 試 科 目	開課系級	考試日期	印製份數	命題教師	備　　　註
工程數學（二）	河海工程學系 2 年級	6 月 10 日		陳桂鴻	

1. $A = \begin{bmatrix} -1 & 1 & -3 \\ 1 & 0 & 2 \\ 2 & -1 & 4 \end{bmatrix}$

(1) Compute $|A|$.　　　3%

(2) Find all eigenvalues and corresponding eigenvector.　　8%

(Hint: only one linear independent eigenvector)

(3) Find generalized eigenvectors and write the transition matrix P of A.　　8%

(4) Find P^{-1}　　3%

(5) Find the Jordan canonical form of A by using the similar transform ($P^{-1}AP$).　　8%

2. Given quadratic form $Q(x) = 3x_1^2 + 2x_1 x_2 + 3x_2^2$.

(a) Write the matrix form $\tilde{x}^T A \tilde{x}$ of $Q(x)$, where $\tilde{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and A is real symmetric matrix.　　3%

(b) Find all eigenvalues and corresponding eigenvectors, and write the transition matrix P of A. 5%

(c) Find P^{-1} (Hint: P is orthogonal matrix)　　3%

(d) Diagonalize A by the similar transform ($P^{-1}AP$).　　5%

(e) Find A^{100}.　　5%

(f) Transform $Q(x)$ to the standard form ($\sum_{i=1}^{2} \lambda_i y_i^2$) by using the principal axis theorem.　　6%

(g) What is the definite property of $Q(x)$? (Negative definite or positive definite)　　3%

10 June 2005

1) $A = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

a) is the matrix A symmetric ? (2 scores)

b)find the eigenvalues of the matrix (hint: one of the eigenvalues is -2, you guess or solve the others)   (4 scores)

c) find three <u>linearly independent</u> eigenvectors associated with the eigenvalues (12 scores) (you must show that <u>they</u> are linearly independent, though they are not unique)

d) find a matrix P such that $P^{-1}AP$ is a diagonal matrix (6 scores) (you must show the result of $P^{-1}AP$)

e) find a matrix Q such that $Q^{-1}A^5Q$ is a diagonal matrix (6 scores) (you must show the result of $Q^{-1}A^5Q$)

2)

$A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1-i \\ 0 & 1-i & 0 \end{pmatrix}$

a) find $\overline{A}$ (the conjugate of A) (3 scores)

b) find A^t (the transpose of A) (3 scores)

c) is the matrix A <u>hermitian</u> or <u>skew-hermitian</u> ? (3 scores)

d) find the eigenvalues of the matrix A (9 scores)

e) find the eigenvectors associated with the eigenvalues (9 scores)

f) find a matrix Z to diagonalize the matrix A (3 scores)